On Samuelson's Analysis of Imperfect Competition

By
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Below, the analysis of imperfect competition given by Paul Samuelson in his *Economics*[1] is applied to the vintage model used in *Oligopolistic Competition*[2]. This model can be interpreted as a description of a large number of firms, each producing the same product using the same production technology. The starting point is the estimation results shown therein for the US manufacturing industry, using data for 1997. However, we assume a stationary economy for that year, with investments in previous years assumed to be exactly equal to actual investment in 1997. This, in deviation from *Oligopolistic Competition*, in which in principle the actual investments are always taken into account for the successive years.

Part A of figure 1 shows the development of revenues and costs as more vintages come into use in 1997. Revenues minus costs determine the profit, which reaches a maximum with the use of vintage 31. In that vintage, marginal costs equal marginal revenues. When even older vintages come into use, total profit decreases because production with these vintages results in a loss. The figure shows that with production using all profitable vintages, the profit share of the total production value is approximately 25 percent.

In *Economics*, Samuelson develops an analysis that distinguishes between fixed and variable costs. Based on this, he derives curves for both marginal unit costs and average unit costs. In line with this, in the vintage model we first look at the fixed and variable costs associated with the new machinery that has just been put into use. The purchase price of this new machinery determines the fixed production costs, which remain unchanged as production increases with the use of older vintages of machines[3]. Variable costs consist of wages for the required labour, which increase as more vintages and increasingly older machines are included in the analysis. On the other hand, there are no fixed costs associated with these previously purchased machines.

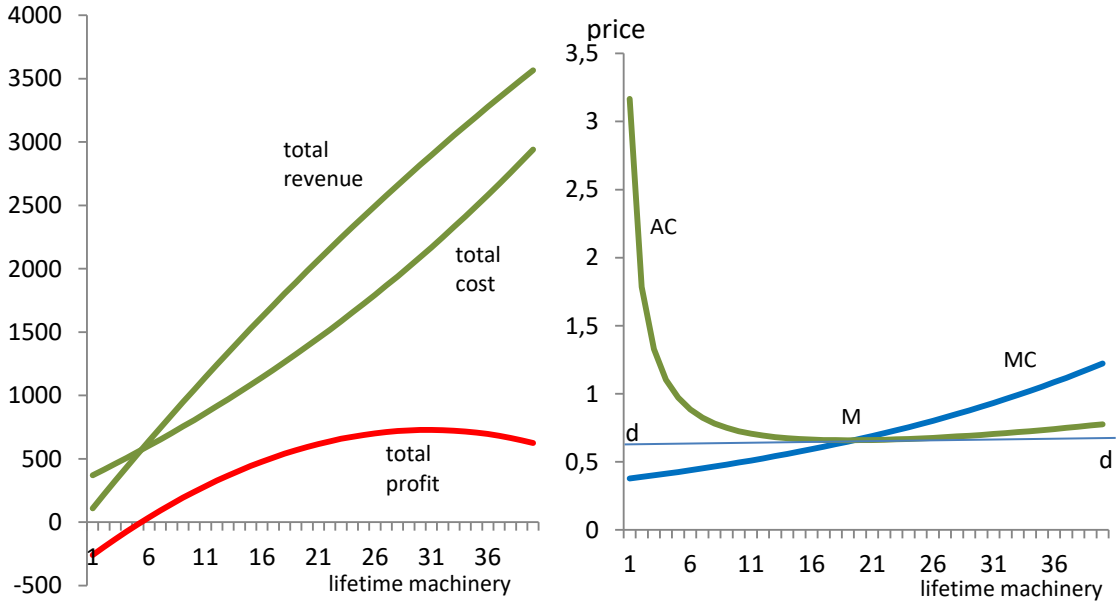
[1] P. Samuelson, *Economics, Ninth Edition*, McGraw-Hill, 1973.
[2] A. Moons, *Oligopolistic Competition and Economic Development*, published on www.davidricardo-firstprinciple.com (to be referred to as *Oligopolistic Competition*).
[3] So we follow Samuelson's table 24-1, op. cit., p. 464.

Part B presents Samuelson's analysis. If only the most recent vintage is considered, the fixed costs associated with purchasing the machinery result in relatively high average unit costs. For this most recent vintage, the marginal costs are significantly lower. However, as more vintages are considered, the marginal costs increase, while the average costs initially decrease, until the marginal costs equal the average unit costs at break-even point M. Or, in Samuelson's words:

> So long as Marginal Cost is below Average Cost, it is pulling Average Cost down; when MC get to be just equal to AC, AC is neither rising nor falling and is at Minimum AC; after MC is above AC, it is pulling AC up.[4]

Part B of figure 1 shows both MC and AC. Samuelson shows a similar figure with the title "Competitors are in long-run equilibrium where Price equals Minimum Average Cost". But does this break-even point M also mean that marginal cost equal marginal revenue? In any case, following Samuelson's example, the figure does show the horizontal demand curve, which the individual firms in the vintage model cannot influence.

Figure 1
Revenue, cost and profit, ranked according to the lifetime of machinery in use
A Results US manufacturing 1997 (million US $) **B Marginal and average cost per unit of production**



The figure also shows that the price at the break-even point is 0.66. However, this is well below the actual production price, which was 0.94 in 1997[5]. Moreover, part B seems to suggest that the economic lifetime is approximately 19 to 20 years, although Samuelson does not yet make a statement on this point in anticipation of his analysis of imperfect competition. Nevertheless, the question remains why this lifetime deviates from the economic lifetime as determined in *Oligopolistic Competition*. How should this be interpreted?

Are there serious shortcomings because the (too) simplistic vintage model does not take into account the intermediate supplies required for production, such as raw materials, energy, etc., nor does it take into account the necessary buildings? If intermediate supplies were to be taken into account, leading to a cost increase of almost 25 percent of the production value, along with 15 percent higher

[4] ibid., p. 468.
[5] In the vintage model, the price at the end of the estimation period 1975-2016 is assumed to be equal to 1.

fixed costs and approximately 5 percent higher variable labour costs, this would lead to a break-even point for average and marginal costs that is exactly equal to the actual production price of 0.94. However, even with such a cost increase, the economic lifetime of machines is only slightly higher than the suggested economic lifetime of approximately 19 years. Moreover, it turns out that only in that one year the profit is not negative. The difference between the calculated price of 0.66 and the actual price should therefore be seen as indicative of the profit achieved in 1997.

To better understand Samuelson's analysis, the top panel of figure 2 shows part B of figure 1 again, but now supplemented in the bottom panel with the profit shown in part A of figure 1, but expressed per unit of production

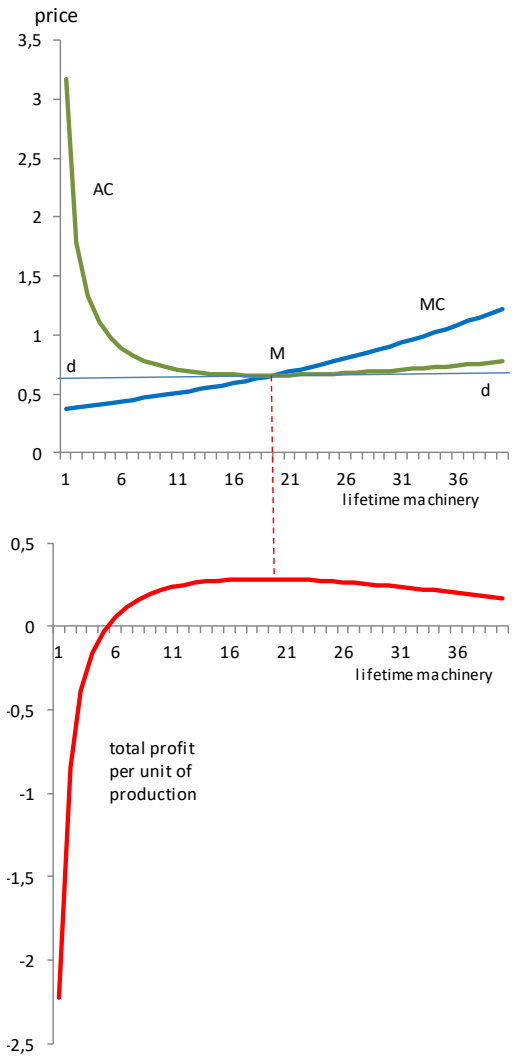
This profit per unit of production reaches a maximum at the breakeven point M at vintage 19/20 and then declines steadily even though total profit reaches a maximum at vintage 31, where marginal revenue equals marginal cost.

We will shortly see how Samuelson's analysis leads us to the most profitable lifetime. But first, let's consider how Samuelson uses part B of figure 1 to illustrate the existence of imperfect competition.

Essentially, Samuelson assumes that firms are so large that they can only sell an increase in production if they lower the price of that production. They therefore face a downward-sloping demand curve that allows them to determine the level of production at which they maximize their profit.

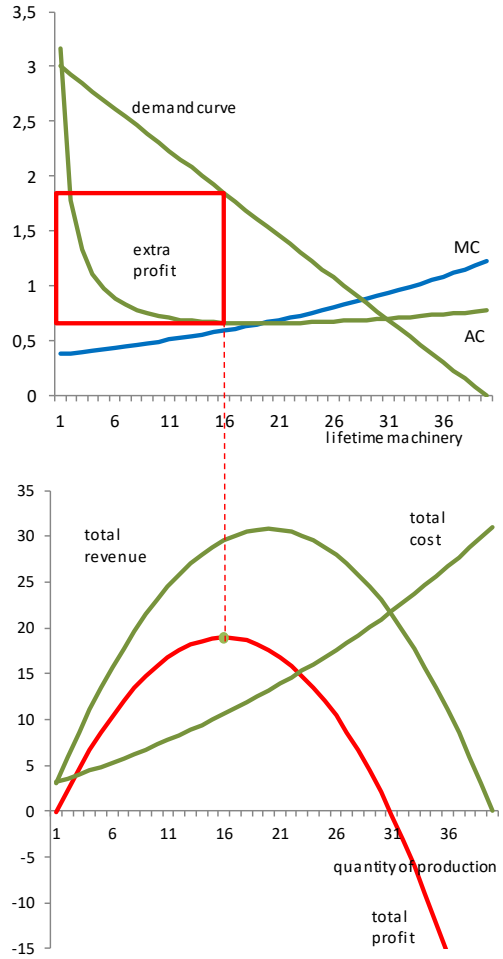
Figure 3 shows this.

Figure 2
Marginal and average cost per unit of production,
In comparison with the profit per unit of production



The vertical axis of the top panel of figure 3 represents the unit price of production. The horizontal axis, along the downward-sloping demand curve, shows the volume of production that can be sold at a given price. It is assumed that production increases at the same rate with the age of vintages.

Figure 3
Profit maximization in case of imperfect competition



The bottom panel of figure 3 shows the revenues from the potential production of a firm using the same production technology as the firms underlying part A of figure 1. Therefore, the unit costs for the firm in figure 3 are the same as those shown in part B of figure 1. Revenues minus costs determine in the bottom panel of figure 3 the profit of the firm, which reaches a maximum at an production of 16 units. The top panel determines the price that can be charged for this production, given the demand curve. The extent to which the price per unit of production exceeds the average unit cost for an production of 16 units determines the profit, which, following Samuelson, is represented in the top panel by the red square.

Samuelson emphasizes that he wants to provide tools "to analyse imperfect as well as perfect competition. They will show what modifications have to be made in any conclusions that were based on an analysis of perfect competition". To fully understand Samuelson's analysis, I would like to provide his brief summary of chapter 25 *Maximum profit equilibrium: Monopoly*.

> Part A of this chapter gives an overview of *patterns of imperfect competition and real-world market structures*. It also present a new important tool – the concept of Marginal Revenue. Part B portrays the equilibrium analysis of an idealized monopoly firm, to show how it *achieves maximization of profit by balancing its Marginal Cost and Marginal Revenue*. We then are ready to appraise at the end of this chapter the inefficiency inherent in imperfection of competition. [6]

We saw how the idealized monopoly firm achieves its maximum profit in the bottom panel of figure 3. But to illustrate the inefficiency inherent in imperfect competition, Samuelson calls "perfect competition a special case of imperfect competition" because the P=MC rule from part B of figure 1 also implies that P=MR. "Here is why: For a small perfect-competitor, Marginal Revenue works out to be exactly the same thing as Price. With no need to cut your P to sell an extra unit of q, the incremental Marginal Revenue it brings you is precisely the P received for that last unit, with no loss

[6] ibid., p. 481.

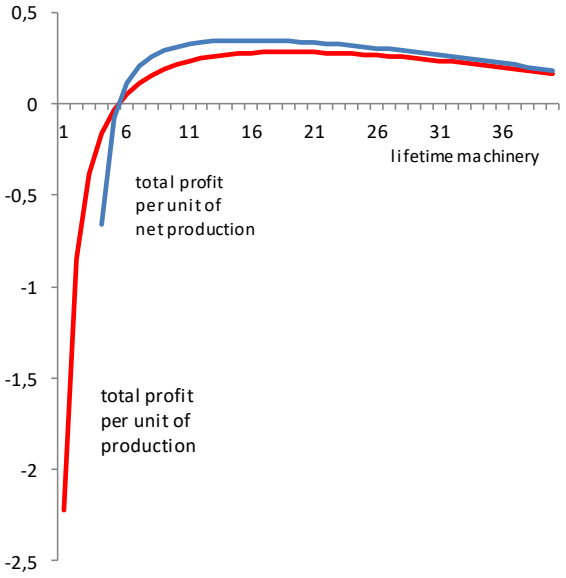
on previous units being subtracted"[7]. P=MR=MC is therefore, according to Samuelson, precisely "at a perfect-competitor's Maximum-profit point".

The inefficiency is then revealed by the extent to which the price under imperfect competition is higher than the Marginal Cost. "The imperfect competitor is contriving to keep things a little scarce. He is contriving to keep P above MC because in that way he sets MR=MC and thereby maximizes his profit. So society does not get quite as much of his good as it really wants in terms of what that good really costs society to produce!"[8].

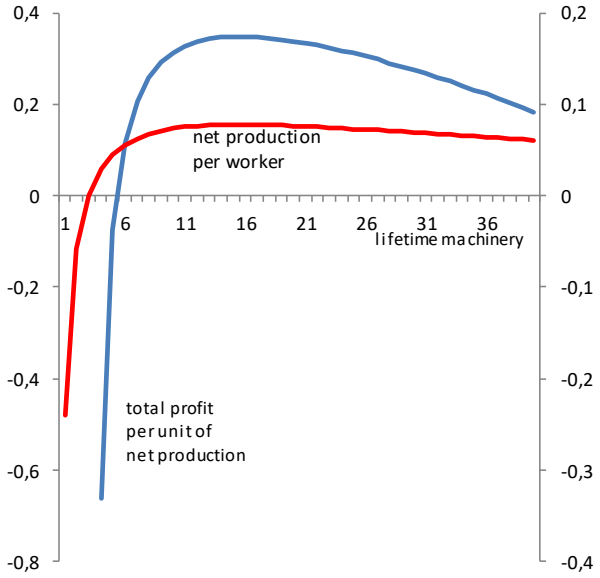
You understand, this raises several questions that become even more pressing as we examine figure 4. Part A of this figure again shows total profit per unit of production, but now supplemented with profit per unit of net production, i.e., production excluding the production of investment goods.

Figure 4
Profit maximization and maximization of net production per worker in case of imperfect competition

A Total profit per unit of production and per unit of net production



B Total profit per unit of net production (left axis) in comparison with net production per worker (right axis)



Part B again shows the profit per unit of net production. But it turns out that the maximum profit per unit of net production is achieved in the same vintage in which the net production per worker reaches a maximum.

What story do the figures presented here tell? While it must first be stated that they are all consistent with each other, I must also recall Joseph Schumpeter's *Preface to the Second Edition* of *Capitalism, Socialism & Democracy*. In it he discusses the reception of the first edition and responds to the "most frequent criticism … directed against what many readers considered to be a defense of monopolistic practice. Yes, I do believe that *most* of the current talk about monopoly, like *all* the current talk about the dire effects of saving, is nothing but radical ideology and has no foundation in fact"[9].

[7] ibid., p. 496.
[8] ibid., p. 498.
[9] J. Schumpeter, *Capitalism, Socialism & Democracy*, Routledge, London, 1943, p. 412.

The main point of his answer, for me, is in line with the usual “comparison between the way in which a purely competitive economy functions and the way in which an economy functions that contains substantial elements of monopoly”, noting that the usual “theory assumes given demand and cost conditions, the same for the competitive and the monopolistic case, whereas it is of the essence of modern big business that its demand and cost conditions are, for large quantities of output, much more favorable – and inevitably so – than the demand and cost conditions that would exist in the same industries in a régime of perfect competition”[10].

In line with this, Samuelson's analysis applied to the vintage model appears to be pulled apart, losing the focus on the equality of marginal revenue and marginal costs, which in the vintage model is expressed in the economic lifetime of the oldest vintage that does not generate a loss. Instead of this focus on Marshallian firms, the analysis seems to be drawn in the direction of more efficient production, so much so that in Samuelson's analysis of imperfect competition, there is no longer less efficient production, but, on the contrary, much more efficient production compared to production achieved under so-called perfect competition, where machines are kept in use until their economic lifetime expires. It seems coincidental, but the idealized monopolistic firm (in my view, a Schumpeterian firm) in the analysis presented here replaces its machines after 16 years, precisely the age identified as the most profitable in *Oligopolistic Competition*. The downward-sloping demand curve could be transformed into a downward-sloping "markup" curve that, with increasingly less efficient production, falls to the price set by Marshallian firms. The markup then indicates the additional profit the Schumpeterian firm can earn by producing increasingly efficient than its Marshallian competitors.

Schumpeter places great emphasis on the need to look carefully at the facts. But the fact that the vintage model is able to pull Samuelson's analysis apart is because this model has the potential to reflect many facts. Be that as it may, I would like to quote Schumpeter again:

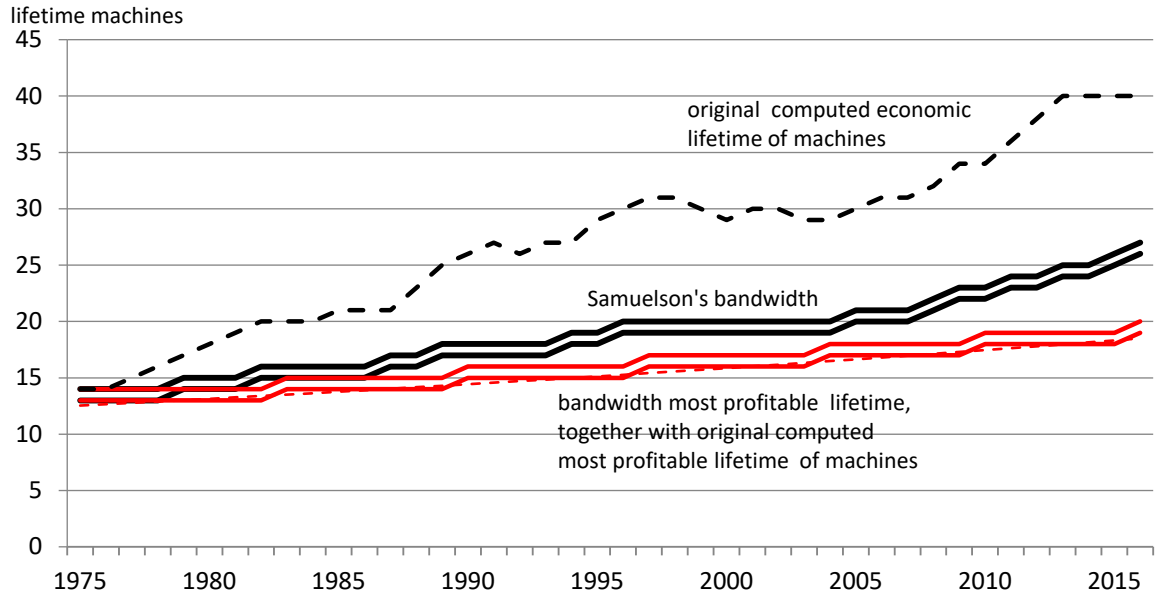
> Is it not entirely futile to elaborate inferences from observed facts without arriving at practical recommendations? I was greatly interested whenever I met with this objection – it is such a nice symptom of an attitude that accounts for much in modern life. We always plan too much and always think too little. We resent a call to thinking and hate unfamiliar argument that does not tally with what we already believe or would like to believe. We walk into our future as we walked into the war, blindfolded. Now this is precisely where I wanted to serve the reader. I did want to make him think. And in order to do so it was essential not to divert his attention by discussions about what from any given standpoint “should be done about it” which would have monopolized his interest. Analysis has a distinct task and to this task I wished to keep though I was fully aware of the fact that this resolve would cost me a great deal of the response a few pages of practical conclusions would have evoked.[11]

In addition to Samuelson's analysis of imperfect competition, figure 6 provides information on the factor-price frontiers as derived using the vintage model for a Marshallian and Schumpeterian firm. But first, figure 5 summarizes the results of the above analysis for the entire estimation period. Samuelson's bandwidth is the bandwidth for the consecutive years in which profit per unit of gross production reaches a maximum. The bandwidth for the most profitable lifetime indicates how net output per worker reaches a maximum over time. This bandwidth aligns well with the most profitable lifetime as determined using the relevant formula included in *Oligopolistic Competition*[12].

[10] ibid., p. 412.
[11] ibid., p. 413.
[12] Op. cit., see formula (9), p. 7.

Figuur 5

Economic lifetime of machinery in US manufacturing industry, together with Samuelson's bandwidth for maximum profit per unit of gross production and a bandwidth for the maximum net production per worker



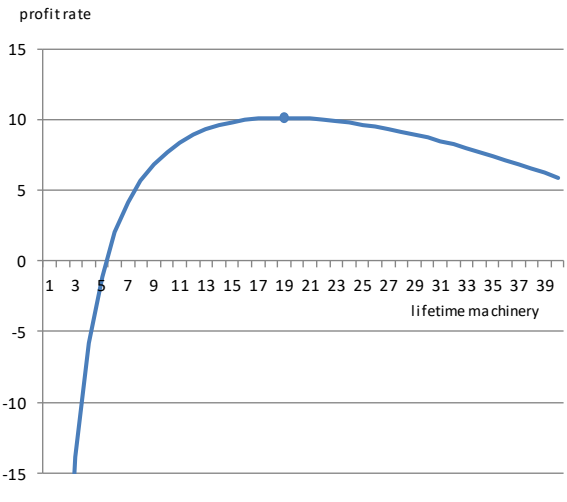
To determine the relationship between the price of labour and the price of capital, the value of capital per vintage is determined based on the replacement value of the capital required to generate the production capacity of successive vintages. Part A of figure 6 shows the profit as a percentage of the value of the capital employed, achieved by using increasingly more vintages. Not surprisingly, the highest profit percentage is achieved when using the youngest 19 vintages.

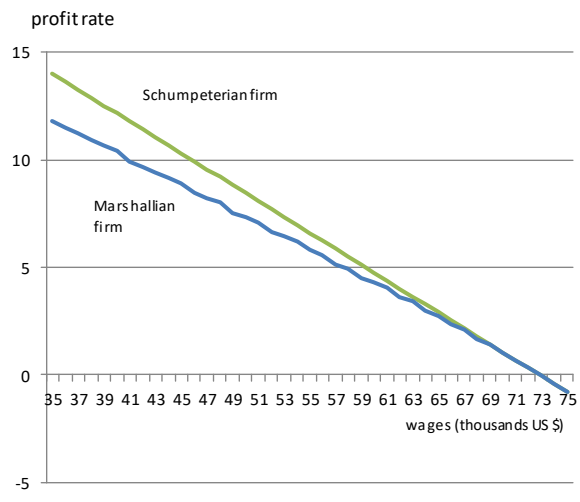
Figure 6

Profit rates on capital together with factor-price frontiers for wages and capital

A Profit rates on capital according to the lifetime of machinery in use (US data 1997)

B Factor-price frontiers for wages and capital of Schumpeterian and Marshallian firms





Part B shows that the profit rate of the Schumpeterian firm (that always replaces its machines after their most profitable lifetime) is higher than the profit rate of the Marshallian firm that shortens the lifetime of its machines as wages rise, until the wage becomes so high that the Schumpeterian firm no longer generates any profit. At that high wage, the economic lifetime equals the most profitable

lifetime. The Marshallian firm then also no longer generates any profit. If the wage rises further and the Schumpeterian firm continues to produce according to its most profitable lifetime, the loss rate of the Marshallian firm becomes slightly less negative than that of the Schumpeterian firm because it keeps fewer vintages in use.

I see in this last point no reason to return to the Cambridge controversy in the theory of capital[13]. I would just like to point out the peculiarity that the marginal product of labour at a wage of almost US$ 46,000 for vintage 31 yields a maximum profit (see part A of figure 1), while at the same time the rate of profit, measured over capital, comes to almost 8.5 percent, as can be seen both from part A and from part B of figure 6. But part A also shows that the rate of profit at the same wage reaches a maximum of just over 10 percent when the machinery is replaced after 19 years.

How should we interpret this? Part B, with the factor-price frontier, is fully consistent with the profit earned by the Marshallian firm by keeping vintages in use as long as they don't incur a loss. That is, as long as marginal costs are less than or equal to marginal revenue. Part A of Figure 6, on the other hand, echoes Samuelson's analysis of marginal and average costs per unit of output. The rate of profit on the value of capital also reaches a maximum here that is, as it were, pulled towards the most profitable lifetime.

[13] G. Harcourt, *Some Cambridge controversies in the theory of capital*, Cambridge University Press, 1972.