Marshallian Production evokes Schumpeterian Production[1]

By

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Oligopolistic competition easily emerges in a world of perfect competition. In an environment with only Marshallian firms that, in producing a specific commodity, all have the same set of possible production and all maximize profits by balancing marginal revenues and marginal costs, Schumpeterian firms may stand up. By starting to produce more efficiently they get market power over Marshallian firms and become, by using their higher profits to expand the set of possible production, forceful drivers of economic development.

This paper first recalls the neoclassical theory. Although it fails to identify efficient production, the theory remains of importance. Just the interaction between Schumpeterian and Marshallian firms helps in explaining why societies fall back in growth at times and have to cope with long lasting unemployment. It enables to disentangle other complexities seen in reality as well and, inter alia, to explain when and why entrepreneurial activity sometimes wait for political action or guidance.

KEYWORDS: economic theory, efficient production, competition, growth and economic development.

1. INTRODUCTION

Oligopolistic competition is usually seen as an example of imperfect competition due to the presumed inherent less efficient production in comparison with perfect competition. Although classifying the real world as neither perfectly competitive nor perfectly monopolistic, Samuelson (1973) stresses that, as soon as we have imperfect competition in the real world, we have left the Garden of Eden: then, we have to deal with how to minimize the inefficiencies involved in such imperfections. However, Samuelson and many others eagerly admit that product differentiation or innovative firms that go ahead in developing new products and production processes could make these inefficiencies less badly.

[1] This paper is a revised version of the paper *On the Emergence of Oligopolistic Competition* that your writer would have liked to present at the EEA-ESEM conference in 2016. Your writer also refers eagerly to his paper *Draft of Equilibrium – In pursuit of efficient production* which explains for a simple economy without capital how, contrary to neoclassical theory, firms can strive for efficient production and therefore maximum profits in a direct competition with each other.

In contrast, this paper shows that oligopolistic competition frequently implies higher, not less efficiency. It shows how oligopolistic competition quite naturally emerges in a market of perfect competition in which all firms have the same developing set of production possibilities in producing the same commodities, year after year. And in which all strive for a maximum profit by equalizing marginal revenues and marginal costs. In such a world it is almost always possible for firms to attain higher levels of efficiency. By their more efficient production these firms get market power over the firms that keep producing in accordance with the principles of the neoclassical theory. The Schumpeterian firms may therefore influence market prices and expand their market shares at the expense of Marshallian firms.

By their higher profits Schumpeterian firms are also especially equipped to come up with "neue combinationen" or otherwise to expand the set of possible production. So, they may further strengthen their position on the market. But how firms succeed in developing production technologies over time is a question to be dealt with here only in passing. The paper primarily focuses on how profit maximization by equalizing marginal revenues and marginal costs can indeed be surpassed by more efficient and more profitable production. It does so by confining the analysis to the existing technology.

The way of thinking bears a slight resemblance to the Solow growth model, consistently seen from a firm's perspective. The literature usually makes a distinction between firms that, independently of time and given the available inputs of labour and capital, maximize their profits on the one hand, and consumers that decide whether to consume now or in the future on the other hand. On an aggregate level higher savings of consumers are needed to enable higher investment and to provide for a higher amount of capital that may induce a steady state with a higher rate of growth and a higher level of efficiency.

The idea is quite simple as soon as one sees that Schumpeterian firms do not have to wait for the higher savings of consumers but may take the lead towards higher levels of efficiency themselves. By their higher profits they may even enforce the savings they need. However, it is not that easy. Marshallian firms do not have the capabilities neither to see nor to attain these higher levels of efficiency. And you too need some of the entrepreneurial capabilities of the Schumpeterian firms yourselves to wrest from the neoclassical principles of equalizing marginal revenues and marginal costs. But mathematics helps, although… By adhering too simple and disjointed mathematical concepts together with too much eagerness to deliver appealing results mathematics in a way also helped the neoclassical theory in moving itself on a dead-end track. A more integrated approach, however, broadens the scope and rolls out new tracks for further progress.

The paper starts to describe the general structure of the economy based upon the set of possible production, conceived as an aggregation of all production processes enabled by (combinations of) the smallest indivisible units of capital, such as a hammer, a truck or a chemical plant. In describing these possibilities we do not have to rely on assumptions that conflict with reality. The only assumption that limits the set of possible production concerns the indispensability of labour.

Section 3 describes Marshallian production and proves the existence of an equilibrium in which no firm can attract labour from another firm and all firms maximize their profits with a marginal product of labour equal to its marginal costs. After an example to clarify why Marshallian production does not necessarily imply efficient production, section 5 dwells more in depth on efficient production. While it turns out to be independent of wages and prices, efficient production could only be defined by taking account of time. This section shows how Schumpeterian firms quite naturally emerge and coexist along with firms that continue their Marshallian production. Next, section 6 deals with oligopolistic competition and the importance of entrepreneurship as a production factor besides capital and labour. The concluding section 7 goes further into some of the economic and societal implications of oligopolistic competition.

Before giving a glance at this broader perspective it must be underlined that capital formation will almost always be below its optimal level. Sometimes, economies seem to suffer from overinvestment and overcapacity with capital, financial capital, abundantly available. But such stages of economic development call for a comparison with two communities separated by a river which economically grow to each other but would really benefit as soon as both are joined by a bridge. As soon as building a bridge becomes part of the production possibilities, firms on both sides of the river may however postpone their investment activities awaiting whether and when the bridge will actually be build. As long as both communities keep talking, firms may hesitate to adapt their investment to the awakening future, while their stagnant investment provides capacity for building the bridge at the same time. Watersheds between communities have many appearances. The present worldwide low real interest rates indicate another watershed far more difficult to solve than just by building a bridge. Consolidation of Schumpeterian firms that induces contraction of Marshallian firms may give rise to long-lasting periods of unemployment and darken passable roads towards sustainable growth. But before putting the finger on the need for governments to take the lead in paving the way for new investments and open new ages of prosperity, we first have to establish some economic fundamentals, based upon the development of the production possibilities and the velocity by which they are exploited.

Further attention to theoretical issues is also required, because the question how the production possibilities develop over time brings back on the agenda the question pointedly put forward by Ricardo: how could we derive from the price formation shown by the markets where, in which specific part of the set of production possibilities changes have occurred? Ricardo kept wrestling with this problem of how to measure the pattern of technical (and technological) change until his dead. The difficulties – arising from the distribution of income between labourers and capital owners, which almost found a solution in his correspondence with Malthus – were, however, prematurely seen as 'solved' when McCulloch introduced his frontrunner of the neoclassical production function. With work to be done section 7 will also close.

2. GENERAL STRUCTURE OF THE ECONOMY

The economy $\mathcal{E}$ consists of firms that produce a finite number of commodities i, $i = 1,\dots,n$, in successive periods of time t. Initially we assume that the firms producing commodity i are identical. The production of commodities takes place in production processes enabled by the capital that firms possess. Each capital structure C_i^t enables a variety of production processes k, $k = 1,\dots,m$, while each production process itself describes the output of commodities in relation to the input of commodities, together with the input of labour, supplied by households. So, capital C_i^t, initially also supplied by households, defines for the production of commodity i up to m production processes $(\lambda_{ki}^t, x_{ki}^t, y_{ki}^t)$ where

λ_{ki}^t is a nonnegative scalar, representing the use of homogenous labour,
x_{kj}^t an n-dimensional nonnegative input vector of intermediate consumption, and
y_{ki}^t an n-dimensional nonnegative output vector.

All technologically possible production processes for commodity i enabled by capital C_i^t build up m production technology sets T_{ki}^t. The assumptions on T_{ki}^t are as follows:

ASSUMPTION 1: *$(\lambda_{ki}^t, x_{ki}^t, y_{ki}^t) \in T_{ki}^t$ and $\lambda_{ki}^t = 0$ imply $y_{ki}^t = 0$. Labour is indispensable.*
ASSUMPTION 2: *T_{ki}^t is compact, which means that T_{ki}^t is closed and bounded.*

ASSUMPTION 3: $y_{ki,kj}{}^t = 0$ *for every kj ≠ ki. No joint production.*
ASSUMPTION 4: $(\lambda_{ki}{}^t, x_{ki}{}^t, y_{ki}{}^t) \in T_{ki}{}^t$ *and* $u_{ki}{}^t \geq x_{ki}{}^t$, $y_{ki}{}^t \geq z_{ki}{}^t \geq 0$ *implies* $(\lambda_{ki}{}^t, u_{ki}{}^t, z_{ki}{}^t) \in T_{ki}{}^t$*. Free disposability.*

These assumptions reflect reality. Compactness results from indispensable labour. As stated by Georgescu-Roegen (1966), compactness has to be assumed if "labour, while an indispensable factor in production of any commodity, can be substituted by other factors beyond any limit". Such a substitution clearly conflicts with reality. The closedness of $T_k{}^t$ implies that if a sequence of possible production processes approaches to $(l_{kx}{}^t, x_{kx}{}^t, y_{kx}{}^t)$ then $(l_{kx}{}^t, x_{kx}{}^t, y_{kx}{}^t) \in T_k{}^t$. Because convexity is not assumed, $T_k{}^t$ may involve increasing returns to scale. The implicit assumption of homogenous labour is not essential. But admitting heterogeneous labour would make the analysis far more complex.

Each capital structure $C_i{}^t$ consists of commodities produced in earlier periods *ts* (*ts = 1,…,s)*, and could be described by a matrix as shown by equation (1). This equation also shows that the capital structure is exposed to wear and tear. In order to maintain the productive capabilities it could be necessary in each period *t* to undo the depreciation $\delta_{ki}{}^{t-ts}$ of capital component $c_{ki}{}^{t-1,t-1-ts}$ by investment $i_{ki}{}^{t-1}$. Disinvestments are also possible. So, the development over time is described by

$$C_i^t = \begin{bmatrix} c_{1i}^{t,t-s} & \cdots & c_{1i}^{t,t-1} \\ \vdots & \ddots & \vdots \\ c_{mi}^{t,t-s} & \cdots & c_{mi}^{t,t-1} \end{bmatrix} = \begin{bmatrix} \delta_{1i}^{t-s} & \cdots & \delta_{kn}^{t-s} \\ \vdots & \ddots & \vdots \\ \delta_{1i}^{t} & \cdots & \delta_{kn}^{t} \end{bmatrix} \begin{bmatrix} c_{1i}^{t,t-1-s} & \cdots & c_{1i}^{t-1,t-2} \\ \vdots & \ddots & \vdots \\ c_{mi}^{t,t-1-s} & \cdots & c_{mi}^{t-1,t-2} \end{bmatrix} + \begin{bmatrix} -i_{1i}^{t-s} & \cdots & i_{1i}^{t-1} \\ \vdots & \ddots & \vdots \\ -i_{mi}^{t-s} & \cdots & i_{mi}^{t-1} \end{bmatrix} \quad (1)$$

The development of capital that represents a hammer is quite simple. But a truck may already consist of over hundred different replaceable commodities. In determining the development of capital over time, the investments also determine to a large extent how the production technology sets develop over time. The union of all sets $T_{ki}{}^t$ defines the overall technology set of the firms that produce commodity *i* at period *t*,

$$T_i^t = \cup_k T_{ki}^t,$$

which consists of production processes

$$(\lambda_i^t, x_i^t, y_i^t) = \sum_{k=1}^m (\lambda_{ki}^t, x_{ki}^t, y_{ki}^t).$$

For reasons of simplicity, we will use from now production process $(\lambda_i{}^t, x_i{}^t, y_i{}^t)$ to represent all possible production processes that belong to the set $T_i{}^t$.

The total technology set of the economy is defined by the union of the sets $T_i{}^t$

$$T^t = \cup_i T_i^t.$$

It should be stressed that all production technology sets are based upon the capital they use. For this capital also holds

$$C^t = \cup_i C_i^t.$$

By supplying firms with labour the households get a wage w^t per unit labour in return. This enables them to buy commodities for an amount of $w^t \lambda^t$. The commodities are priced with price vector p^t. The firms that hires labour to perform production process $(\lambda^t, x^t, y^t) \in T^t$ will make a profit equal to $p^t (y^t - x^t) - w^t \lambda^t$. Because T^t is compact this profit has a maximum.

In less advanced stages of an economy households also provide firms with capital C^t. The firms pay the households an average profit rate over the value of the capital they use. In competitive markets the supply of both labour and capital will be rewarded by wages and profit rates that are the same in all firms.

To determine the profit rate, it is sufficient to look at the production processes that use brand new capital. Because capital can be used flexibly here, competition will ensure that every use of capital is equally profitable. So, for each production process $(\lambda^t, x^t, y^t) \in T^t$ the profit rate follows from

$$\pi^t_{rate} = \frac{\sum_i (p^t(y^t_{1i} - x^t_{1i}) - w^t \lambda^t_{1i})}{\sum_i p^{t-1} i^{t-1}_{1i}}. \qquad (2)$$

The value $V^{new,t}$ of this capital will thus be equal to $\sum_i p^{t-1}\, i_{1i}^{t-1}$. The profit rate makes it possible to determine the value of the previously installed capital

$$V^{old,t} = \frac{\sum_{i=1}^{n} \sum_{k=2}^{m} (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki})}{\pi^t_{rate}}. \qquad (3)$$

This value will reflect the technological progress that newly produced capital may bring about. More efficient production with less input of labour or less intermediate inputs to produce one unit of output will make more recently installed capital more profitable. This can be reflected in a higher rate of profit, but often also in a lower value of previously installed capital. This value eventually drops to zero as soon as capital can no longer produce profitably.

For each production technology set T_i^t we define a set of wage-price vectors P_i^t that enable profitable production:

$$P_i^t = \{(w^t, p^t) \in R^{1+n} | \exists (\lambda^t_{ki}, x^t_{ki}, y^t_{ki}) \in T_i^t : \ (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki}) \geq 0\}$$

The set wage-price vectors $P_s{}^t$ that enable all firms to produce with its production processes in a profitable way follows from the intersection

$$P^t = \cup_i P_i^t.$$

Conversely, by having $P_i{}^t$ we are also able to restrict the sets T_i^t, which consists of the production processes as we have seen earlier

$$(\lambda_i^t, x_i^t, y_i^t) = \sum_{k=1}^{m} (\lambda^t_{ki}, x^t_{ki}, y^t_{ki}),$$

to sets of profitable production processes only. So, we define

$$T^t_{i+} = \{(\lambda_i^t, x_i^t, y_i^t) \in T_i^t | \exists (w^t, p^t) \in P_i^t \ and \ \exists k : \ \ (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki}) \geq 0\}$$

Moreover, within the compact sets $T_{i+}{}^t$ we could define the production processes that provide a maximum profit for any allowed vector (w^t, p^t). So,

$$T^t_{i\,MAX} = \{(\lambda_i^t, x_i^t, y_i^t) \in T^t_{i+} | \exists (w^t, p^t) \in P_i^t : \ MAX(\sum_k (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki}))\}$$

The total set T_+^t of profitable production within the economy follows from the union of the sets T_{i+}^t. For each vector $(w^t, p^t) \in P^t$ there exists a profit rate $\pi_{rate}{}^t$ determined by formulae (2). So, we also have an extended set P_s^t which contains the vectors $(w^t, \pi_{rate}{}^t, p^t)$ for all $(w^t, p^t) \in P^t$.

In addition we discern the set $Y_{i+}{}^t$ of the net output of commodity *i* that firms could produce profitable with the available labour:

$$Y^t_{i+} = \{(y_i^t - x_i^t, \lambda_i^t) | \sum_k (\lambda^t_{ki}, x^t_{ki}, y^t_{ki}) \in T^t_{i+}\}.$$

The union of all sets $Y_{i+}{}^t$ defines the set $Y_+{}^t$ describing the net output of commodities that could be produced profitably by the economy with total labour input λ^t. The economy $\mathcal{E}$ is called productive if the profitable net output of $Y_+{}^t$ has a positive value for at least one commodity *i*.

The commodities that could be produced with the input of labour on a profitable way, should meet the demand for those commodities from the households deriving income from their supply of labour and possibly also of capital. So, for any combination of w^t and $\pi_{rate}{}^t$ the households get an income equal to $w^t \lambda^t + \pi_{rate}{}^t V^t$ in return for their supply of labour and capital in period *t*.

The possible choices of the households with regard of demand and supply are given by

$$X_d^t = \{(d^t, \lambda^t) \in R^{n+1} \big| \forall (w^t, \pi_{rate}^t, p^t) \in P_s^t: \ \ p^t d^t \leq w^t \lambda^t + \pi_{rate}^t V^t\}$$

This set includes for any vector $(w^t, \pi_{rate}^t, p^t) \in P_s^t$ the preferred demand for commodities and supply of labour that gives household *h* the greatest satisfaction. For reasons that become clear in the proof of theorem 1 we add another assumption which does not conflict with reality either.

ASSUMPTION 5: *No commodity is indispensable. Above a certain price level the preferred demand for each commodity i will become zero.*

Let γ_h be the function that according to Debreu[2] enables to identify for any $(w^t, \pi_{rate}^t, p^t) \in P_s^t$ the most preferred combination (d^t, λ^t). Then we could define

$$X_{d+}^t = \left\{(d^t, \lambda^t) \in X_d^t \middle| \forall (w^t, \pi_{rate}^t, p^t) \in P_s^t: \ MAX\left(\gamma_h(d^t, \lambda^t)\right)\right\}.$$

The corresponding set of wage-profit-price vectors follows from the inverse of γ_h. So,

$$P_d^t = \left\{(w^t, \pi_{rate}^t, p^t) \in P_s^t \middle| \forall (d^t, \lambda^t) \in X_{d+}^t: \ MAX\left(\gamma_h^{-1}(w^t, \pi_{rate}^t, p^t)\right)\right\}$$

The close relation between P_d^t and P_s^t secures that demand and supply could meet each other. So, people who are only willing to offer labour against exceptional prices are excluded from participation in the economy, unless they are involved in, for instance, entertainment production processes for which other households are willing to pay those exceptional prices.

3. MARSHALLIAN PRODUCTION

The sets of both total preferred demand X_{d+}^t and total profitable supply of commodities Y_+^t, as defined for any vector $(w^t, \pi_{rate}^t, p^t) \in P_d^t \cap P_s^t$, suggest that only those vectors (w^t, π_{rate}^t, p^t) for which the intersection of the corresponding demand and supply sets is not empty, could possibly equilibrate supply and demand.

Before proving the existence of such equilibrium we first describe the profit maximizing behaviour in more detail. The profits of firms that produce one commodity may deviate from the profits of firms in producing other commodities. Both the profit rate on new capital and the value of old capital could hardly be determined in reality. So, the owners of capital have to rely on what the markets think that the profit rate would be. They may than contribute to an equilibrating process by providing relatively more capital to the more profitable firms. From the perspective of the firms, however, it may be more relevant to look at the differences in profits provided by their least profitable production processes. If those profits vary from one commodity to another, these production processes would clearly not be stable according to the neoclassical theory. The marginal product of labour in producing one commodity would then under the prevailing wage-price vector exceed the marginal product of labour in producing another commodity. To describe this in more detail we first define *profit maximizing firms*.

DEFINITION 1
A firm that, under the prevailing wage-price vector, uses each production process it possesses up to

[2] Op. cit., p. 65 and next.

its maximum profit potential is called a profit maximizing firm.

Next, we assume that the profit maximizing firms producing commodity *i* will expand their production and so their profits, as long as their marginal product of labour MPL_i^t is higher than the prevailing wage rate. This marginal product could be derived within the set $T_{i\,MAX}^t$ from a comparison with the profit maximizing production $y_i^t - x_i^t$ in view of the prevailing vector (w^t, p^t) and the profit maximizing production $y'^t_i - x'^t_i$ which results under the same wage-price vector in case of the available labour is diminished by one unit, for instance one hour. So, the marginal product of this unit labour is defined by

$$p^t(y_i^t - x_i^t) - w^t\lambda_i^t = p^t(y_i'^t - x_i'^t) - \left(w^t\lambda_i^t - MPL_i^t\right)$$

which implies

$$MPL_i^t = p^t\left(y_i^t - y_i^{'t} - \left(x_i^t - x_i^{'t}\right)\right)$$

As long as MPL_i^t exceeds w^t the production processes of commodity *i* could expand their production by taking over labour from other production processes. At the same time the prevailing wage-price vector will continuously adjust to the changing division of labour. So, a convergence process emerges in which MPL_i^t approaches w^t for each production technology *i*.

However, the marginal product of labour is not easily observable. In the reality we see firms in case of relatively high profits striving for even higher profits by taking over labour from other firms. Therefore, we also look to the least profitable production processes in producing each commodity *i* as can be determined for each vector $(w^t, p^t) \in P_i^t$ within the set $T_{i\,MAX}^t$. That means

$$LPT_{i\,MIN}^t = \left\{\left(\lambda_i^t, x_i^t, y_i^t\right) \in T_{i\,MAX}^t \middle| \forall (w^t, p^t) \in P_i^t \;\; and \;\; \exists k: \;\; MIN\left(p^t\left(y_{ki}^t - x_{ki}^t\right) - w^t\lambda_{ki}^t\right)\right\}.$$

This set facilitates the description of the convergence of the marginal product of labour towards an equilibrium wage. We therefore use the following lemma.

LEMMA 1

The marginal product of labour MPL_i^t approaches an equilibrium wage w^t if and only if the profit of the least profitable production processes $(\lambda_{ki}^t, x_{ki}^t, y_{ki}^t) \in LPT_{i\,MIN}^t$ in producing each commodity *i* approaches to zero.

Proof:
In order to show that $MPL_i^t \to w^t$ if and only if profit $(p^t\,(y_{ki}^t - x_{ki}^t) - w^t\,\lambda_{ki}^t) \to 0$ for the production processes belonging to the set $LPT_{i\,MIN}^t$ we firstly show that $w^t = MPL_i^t$ implies a profit equal to zero. If, in case of adding units of labour, the profit remains higher than zero, this would imply some undistributed profits. This contradicts the definition of the marginal productivity of the last added labour unit which in the set $LPT_{i\,MIN}^t$ cannot be higher than wage w^t.

Secondly, a profit equal to zero must imply $MPL_i^t = w^t$. This equality results from the fall to zero in the value of the capital that accompanies the falling profit. If additional units of labour would induce prices that enhance the value of the capital, than wage w^t must become lower than the marginal productivity of labour in order to have a profit equal to zero. This also contradicts the definition of marginal productivity.
Q.E.D.

Lemma 1 helps describe stable production by the following definitions.

DEFINITION 2
Under the prevailing wage-price vector *(w^t, p^t)* the profit maximizing firms that possess the production processes of commodity *i* can take over labour from the firms that produce commodity *j*, *j ≠ i*, if for their least profitable production processes $(\lambda_{ki}^t, x_{ki}^t, y_{ki}^t) \in LPT_{i\,MIN}^t$ and $(\lambda_{kj}^t, x_{kj}^t, y_{kj}^t) \in LPT_{j\,MIN}^t$, holds that $(p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t) > (p^t(y_{kj}^t - x_{kj}^t) - w^t\lambda_{kj}^t)$.
DEFINITION 3
Economy $\mathcal{E}$ is called stable in relation to a wage-profit-price vector if under this vector no group of profit maximizing firms producing one commodity can take over labour from another group producing another commodity.
DEFINITION 4
A transaction $p^t d^t = w^t \lambda^t + \pi_{rate}^t V^t$ is called a Marshallian equilibrium if
1. the economy is stable in relation to the wage-profit-price vector (w^t, π_{rate}^t, p^t), and
2. the preferred demand vector d^t is nonnegative and belongs to the set T_+^t of all profitable net output vectors.

The clarity of the concepts that define the set of profitable production, the subset with maximum profits and within this subset the set of the least profitable production processes does already image how these sets in interaction with a changing wage-price vector evolve towards a Marshallian equilibrium. The following theorem, known as the theorem of Kuratowski, Knaster and Mazurkiewicz (see Berge (1965) and Kuratowski (1966)), accounts for the core of the proof that a Marshallian equilibrium exists indeed.

If $A_0,\ldots,A_n$ are (n+1) closed sets in simplex S such that each face $p_{i0}\ldots p_{ik}$ $(0 \le k \le n)$ of the simplex satisfies

$$p_{i0}\ldots p_{ik} \subset A_{i0} \cup \ldots \cup A_{ik}$$

then $A_0 \cap \ldots \cap A_n = \emptyset$.

A corollary of this theorem got an earlier application in the preliminary proof of fair net trades by Schmeidler and Vind (1970), although they changed their proof later on (see Schmeidler and Vind (1972).

THEOREM 1 (Marshallian equilibrium)

Let economy $\mathcal{E}$ be productive. Then there exists a Marshallian equilibrium $p^t d^t = w^t \lambda^t + \pi_{rate}^t V^t$ in which w^t equals the marginal product of labour within each group of firms producing one of the commodities *i* of the economy, and in which the profit of each group equals $\pi_{rate}^t V_i^t$.

Proof:
The equilibrium transaction must be found in both $Y^{*t} = Y_+^t \cap X_{d+}^t$ and $P_s^{*t} = P_s^t \cap P_d^t$. Both sets are not empty because the economy is productive. From P_s^{*t} we derive P^{*t} by leaving π_{rate}^t out of consideration. Because Y^{*t} could be decomposed in *n* sets Y_i^{*t} which consists of *k* production processes that produce commodity *i* we could define

$$T_{i+}^{*t} = \{(y_i^t - x_i^t, \lambda_i^t) \in Y_i^{*t} \mid \forall (w^t, p^t) \in P^{*t} \ and \ \forall k: \ (p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t) \ge 0\}$$

$$T_{i\,MAX}^{*t} = \{(y_i^t - x_i^t, \lambda_i^t) \in T_{i+}^{*t} \mid \forall (w^t, p^t) \in P^{*t}: \ MAX(\textstyle\sum_k (p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t))\}$$

And

$$LPT_{i\,MIN}^{*t} = \{(y_i^t - x_i^t, \lambda_i^t) \in T_{i\,MAX}^{*t} \mid \forall (w^t, p^t) \in P^{*t} \ and \ \exists k: \ MIN\ (p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t)\}.$$

In order to consider whether the wage-price vectors in P^{*t} support stable production, we define for the least profitable production process within $LPT_{i\,MIN}^{*t}$ of each commodity *i* the set

$$M_i^t = \{(w^t, p^t) \in P^{*t} | \exists\, (y_i^t - x_i^t, \lambda_i^t) \in LPT_{i\,MIN}^{*t} \text{ and } \forall j \neq i\ \exists (y_j^t - x_j^t, \lambda_j^t) \in T_{j+}^{*t} : \ (p^t(y_{kj}^t - x_{kj}^t) - w^t \lambda_{kj}^t) \geq (p^t(y_{ki}^t - x_{ki}^t) - w^t \lambda_{ki}^t)\}.$$

The wage-price vectors of transactions that are a Marshallian equilibrium have to belong to the intersection of all M_i^t. Before proving that $\cap_i M_i^t$ is nonempty, we first show that $\cup_i M_i^t = P^{*t}$.

By the compactness of T_{i+}^{*t} the profit function of $T_{i\,MAX}^{*t}$ reaches a maximum for every (w^t, p^t) supporting T_{i+}^{*t}. Equally, each profit maximizing production has always a least profitable production process. Because $\cup_i T_{i+}^{*t} = Y^{*t}$, we know that the intersection of all M_i^t equals P^{*t}.

Next we embed P^{*t} in the simplex S,

$$S = \{s \in R^n | s = (s^i), \sum_{i=1}^n s^i = 1, \forall i : s^i \geq 0\},$$

with S_i the i^{th} face for each $i \in I$. So,

$$S_i = \{s \in S | s^i = 0\}.$$

Let h be the transformation of P^{*t} into S such that for each $(w^t, p^t) \in P^{*t}$

$$h(w^t, p^t) = \left[\sum_{i=1}^n (p^t(y_i^t - x_i^t) - w^t \lambda_i^t)\right]^{-1}.$$

This transformation is continuous, because the function h' from P^{*t} to R^l such that $h'(w^t, p^t) = (p^t(y_i^t - x_i^t) - w^t \lambda_i^t)$, is continuous. It can also easily be seen that $h^{-1}(S) = P^{*t}$, because by the assumption of free disposability d^t is admitted to be less than d^{*t} so that for one or more i, but not all, $(p^t(y_i^t - x_i^t) - w^t \lambda_i^t) \to 0$ if $(y_i^t - x_i^t) \to 0$.

Next we transform all subsets M_i^t of P^{*t} into S. The closedness of M_i^t implies that $h(M_i^t)$ is closed in S. Moreover, $h(M_i^t) \supset S_i$ because $(p^t(y_{ii}^t - x_{ii}^t) - w^t \lambda_{ii}^t) \geq (p^t(y_i^t - x_i^t) - w^t \lambda_i^t)$ for each $ii \neq i$, while assumption 6 enables that $(p^t(y_i^t - x_i^t) - w^t \lambda_i^t) \to 0$ if $(y_i^t - x_i^t) \to 0$. Therefore, we have for each commodity i a closed set which includes S_i. Because the intersection of these closed sets is equal to S, we know by the theorem of Kuratowski, Knaster and Mazurkiewicz that $\cap_{i \in I} h(M_i^t) \neq \emptyset$.

Next we prove that within a Marshallian equilibrium wage w^t equals the marginal productivity MPL_i^t of the least profitable production processes producing each commodity. By lemma 1 we know that $MPL_i^t \to w^t$ if and only if the profit $(p^t(y_k^t - x_k^t) - w^t \lambda_k^t) \to 0$ for all production processes belonging to the set LPT_{iMAX}^t. This wage w^t must be equal to the equilibrium wage for all the least profitable production processes producing all commodities. Otherwise, there would be a contradiction with the stability requirement.

Finally, the profits of all firms that produce commodities i with brand new capital only, have to determine an equal profit rate π_{rate}^t over the value of their capital. A not equalizing profit rate would clearly conflict with the stability requirement. The production processes with brand new capital only will also be the least profitable ones. So, if the profit potential of such a firm is not fully exploited, it can take over labour from another firm. Otherwise, the household could benefit from a redistribution of their supply of capital to the firms.
Q.E.D.

The proof of existence of a Marshallian equilibrium among each group of firms producing one of the commodities of the economy, evidently also implies the existence of an equilibrium within groups of firms producing similar commodities, shoes for instance, that are distinguishable from firm to firm. However, we are also able to define *Marshallian production* that is related to the individual firms of each group. This will help to describe firm behaviour in more detail, more independent of the suppliers of capital.

DEFINITION 5

Firms that maximize profits under the prevailing wage-price vector *(w^t, p^t)* are called Marshallian firms if they keep their production processes, as defined by the capital they possess, in operation as long as the marginal product of labour on this capital is higher or equal to wage w^t. The production of these firms is called Marshallian production.

Evidently, Marshallian production is a sine qua non of a Marshallian equilibrium and implies a kind of weak efficiency, as we will see more clearly later on. However, many transactions that are clearly not Marshallian equilibria could still involve Marshallian production. Especially in such situations the following theorem is relevant to identify weak efficiency in describing firm behaviour.

THEOREM 2 (Marshallian production)

If under the prevailing wage-price vector *(w^t, p^t)* all labour demand can be met, no firm can attain higher levels of profitable production with the capital it possesses, than the levels defined by Marshallian production.

Proof:

For any given a wage-price vector *(w^t, p^t)* $\in P_i{}^t$ the set $T_{i+}{}^t$ contains all profitable production processes *k*, made possible by capital C_i^t that belongs to the firms producing commodity *i*. Within this set we have defined the set $T_{i\,MAX}{}^t$ which contains the single production processes *k* that provide a maximum profit. And within $T_{i\,MAX}{}^t$ the least profitable production processes are defined by

$$LPT^t_{i\,MIN} = \{(\lambda^t_i, x^t_i, y^t_i) \in T^t_{i\,MAX} | \forall (w^t, p^t) \in P^t_i \;\; and \;\; \exists k: \;\; MIN\; (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki})\}.$$

Evidently, the production processes in which the marginal product of labour equals, or almost equals wage w^t, belong to the set $LPT_{i\,MIN}{}^t$. The maximum profit of the Marshallian production $MP_i{}^t$ is defined by the summation of all maximum profits provided by the production processes *k* that are allowed by capital C_i^t. So,

$$MP^t_i = \sum_{k=1}^{m} (p^t(y^t_{ki} - x^t_{ki}) - w^t \lambda^t_{ki} | (\lambda^t_{ki}, x^t_{ki}, y^t_{ki}) \in T^t_{i\,MAX}).$$

By using all its capital C_i^t, the firms producing commodity *i* cannot attain a higher profit than MP_i^t, and by consequence also not a higher level of production.
Q.E.D.

We need these concepts of Marshallian firms and Marshallian production in order to focus on firms that provide labourers with wage income and capital owners with capital income. Decisions how to maintain existing capital or how to invest in new capital are not necessarily reserved to the owners of capital only, but may indeed also be made by the firms themselves. Therefore, we are going to analyse firms in their capacity to provide capital owners with a capital income that they can just use to consume commodities as wage earners use their income for consumption.

4. INTERMEZZO

In the less advanced economies that we have analysed so far capital formation only depends on households that decide whether to consume or to invest their resources. Firms do not play an active role: they just lend all the capital they need from households. Capital that consists of commodities bought by households, including the commodities with which households extend or maintain the capital they have already lent to firms. Therefore, it is hardly necessary to make a distinction between commodities bought by households for investment or consumption purposes.

In more advanced economies firms start to operate more independently of households, with more complex financing structures that may consist of both bonds provided by households and shares by which household may benefit from more risky firm activities. But more importantly, firms may start to determine their own investment strategies. Evidently, they may pursue similar strategies as performed by households: they may be willing to use earlier installed capital as long as it provides a non-negative profit. So, Marshallian production may still prevail, but – as we will see – firms may also conduct a more profitable investment strategy.

A simple two-sector economy unveils the core of what we call Schumpeterian production. The first sector produces only investment goods. Initially, it produces each year the same amount of machinery with the same 10 units of labour, without using any capital. The second sector only produces consumption goods with capital delivered from the first sector. In the first year of its operation machinery requires 5 units of labour to produce consumption goods with a value of 8. As machinery becomes older each year an additional labour input of 0.2 units is needed to produce the same production value. Moreover we assume that one unit of labour is rewarded by consumption goods with a value of 1. Table 1 describes the production technology set of the second sector with all production processes set out in rows according to the age of their capital.

Table 1 Production technology of consumption sector

AVAILABLE PRODUCTION PROCESSES BY AGE OF CAPITAL								
Age of capital	Capital value by age	Production value	Labour costs	Profit (=3-4)	Profit rate (=5/2)	Capital replacement reserve	Capital income (=5-7)	Capital income rate (=8/2)
1	2	3	4	5	6	7	8	9
0	10.00	8	5	3	0.3	1.25	1.75	0.175
1	9.33	8	5.2	2.8	0.3	1.17	1.63	0.175
2	8.67	8	5.4	2.6	0.3	1.08	1.52	0.175
3	8.00	8	5.6	2.4	0.3	1.00	1.40	0.175
4	7.33	8	5.8	2.2	0.3	0.92	1.28	0.175
5	6.67	8	6	2	0.3	0.83	1.17	0.175
6	6.00	8	6.2	1.8	0.3	0.75	1.05	0.175
7	5.33	8	6.4	1.6	0.3	0.67	0.93	0.175
8	4.67	8	6.6	1.4	0.3	0.58	0.82	0.175
9	4.00	8	6.8	1.2	0.3	0.50	0.70	0.175
10	3.33	8	7	1	0.3	0.42	0.58	0.175
11	2.67	8	7.2	0.8	0.3	0.33	0.47	0.175
12	2.00	8	7.4	0.6	0.3	0.25	0.35	0.175
13	1.33	8	7.6	0.4	0.3	0.17	0.23	0.175
14	0.67	8	7.8	0.2	0.3	0.08	0.12	0.175
15	0	8	8	0	0	0	0	0
Σ 0-15	80	128	104	24	0.3	10.00	14	0.175

Column 2 of table 1 shows the decreasing value of the invested capital as could be derived from the value of brand new capital according to the equations (2) and (3). For instance, capital of one year old gets a value 9.33 (=((8-5.2)/0.3). After 15 years of operation the capital value becomes zero. So, if the consumption sector both adheres the principles of Marshallian production and enjoys a sufficient supply of labour, it will replace machinery after 16 years of operation. In case of a yearly depreciation of 12.5 per cent of the value of its capital total depreciation – the 'capital replacement reserve' of table 1 – allows a yearly investment of machinery with a value of 10. Marshallian production results by this investment in a steady state as shown in the last row of table 1 with a capital income of 14 for the capital owners.

The steady state, as shown by the last row of table 1, is showed again by the last row of table 2. This last row shows that capital will be replaced after 15 year. In the penultimate row the replacement of capital takes place after 14 year. And so on. Table 2 gives alternative production strategies characterized by the same yearly investment but with varying age of replacement of capital. The comparison of steady states in table 2 shows that Marshallian production results indeed in the highest level of profitable production – as stated by theorem 2 – as long as labour is not a binding factor.

In case of a steady state with an earlier replacement of capital the depreciation rate of capital needs to be higher in order to be able to invest the yearly amount of 10 that is needed for maintaining the steady state. By these relatively higher investment levels total profit and capital income diminish the more as the machinery is replaced earlier. The capital income rate becomes even negative in case of a very fast replacement.

Table 2 Steady states, all with a yearly fixed investment of 10, while the age of replacement of capital varies from 1 to 16 year

SECTOR 2: PRODUCTION PROCESSES (TABLE 1) ADDED UP TO AGE OF CAPITAL REPLACEMENT									
Age of capital replacement	Total capital value	Total production value	Total labour costs	Total profit (=3-4)	Profit rate (=5/2)	Depreciation rate	Capital replacement reserve	Total capital income (=5-8)	Capital income rate (=9/2)
1	2	3	4	5	6	7	8	9	10
0									
1	10.00	8	5	3	0.3	1.000	10	-7.00	-0.700
2	19.33	16	10.2	5.8	0.3	0.517	10	-4.20	-0.217
3	28.00	24	15.6	8.4	0.3	0.357	10	-1.60	-0.057
4	36.00	32	21.2	10.8	0.3	0.278	10	0.80	0.022
5	43.33	40	27	13	0.3	0.231	10	3.00	0.069
6	50.00	48	33	15	0.3	0.200	10	5.00	0.100
7	56.00	56	39.2	16.8	0.3	0.179	10	6.80	0.121
8	61.33	64	45.6	18.4	0.3	0.163	10	8.40	0.137
9	66.00	72	52.2	19.8	0.3	0.152	10	9.80	0.148
10	70.00	80	59	21	0.3	0.143	10	11.00	0.157
11	73.33	88	66	22	0.3	0.136	10	12.00	0.164
12	76.00	96	73.2	22.8	0.3	0.132	10	12.80	0.168
13	78.00	104	80.6	23.4	0.3	0.128	10	13.40	0.172
14	79.33	112	88.2	23.8	0.3	0.126	10	13.80	0.174
15	80.00	120	96	24	0.3	0.125	10	14.00	0.175
16	**80.00**	**128**	**104**	**24**	**0.3**	**0.125**	**10**	**14.00**	**0.175**

In this table and also in table 3 it is assumed that machinery, once replaced, has no longer any value, even if the machinery still allows profitable production. Selling still profitable machinery to other firms will transform each steady state into the Marshallian steady state of the last row.

However, in table 3 we concentrate on steady states that are able to produce each year the same amount of consumption goods. Both the depreciation rate and the profit or the capital income rate show no difference from table 2, but total capital income reaches a maximum of 17.6 in case machinery is replaced after 10 years of operation as shown by column 8. With this investment strategy the two-sector economy could reproduce itself not only with a maximum capital income but also with a minimum of required labour:16 units in sector 1 and 94 in sector 2.

All other steady states require more labour input to produce a consumption value of 128 and deliver lower capital income. The Marshallian steady state requires 114 unit of labour input to generate a net profit of 14. Although its capital income rate of 17.5 percent is higher than the rate of 15.7 per cent that arises in the most profitable steady state. This illustrates that as long as labour is not a binding factor, Marshallian production remains profitable. But, as we will see, this does not prevent Schumpeterian firms to benefit from the wage rate determined by Marshallian production and to realize higher capital income by producing more efficiently.

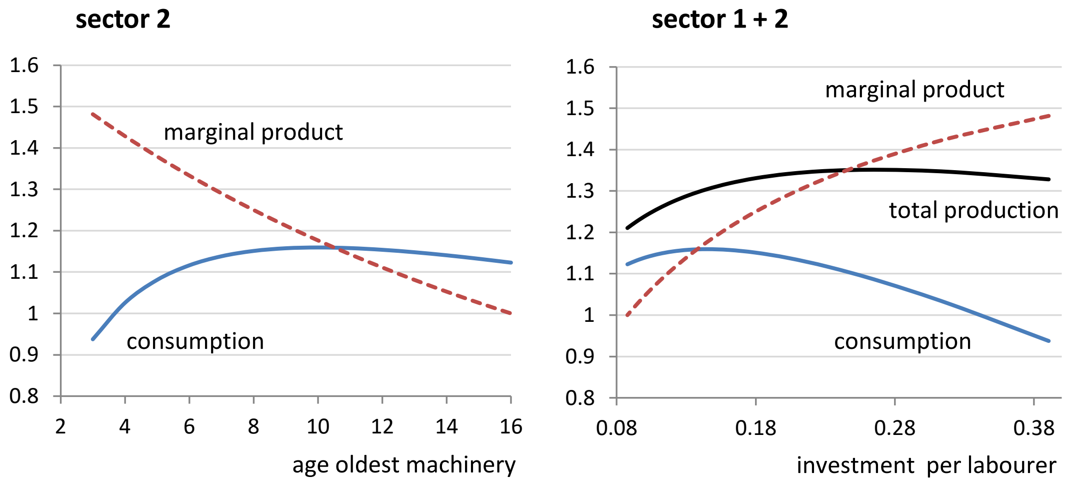
Table 3 Steady states with fixed production and varying investment levels

SECTOR 2: PRODUCTION PROCESSES BY AGE OF CAPITAL REPLACEMENT WITH FIXED PRODUCTION									
Age of capital replacement	Total capital value	Total production value	Total labour costs	Total profit (=3-4)	Profit rate (=5/2)	Depreciation rate	Capital replacement reserve	Total capital income (=5-8)	Capital income rate (=9/2)
1	2	3	4	5	6	7	8	9	10
0									
1	160.00	128	80	48	0.3	1.000	160.00	-112.00	-0.700
2	154.67	128	82	46.4	0.3	0.517	80.00	-33.60	-0.217
3	149.33	128	83	44.8	0.3	0.357	53.33	-8.53	-0.057
4	144.00	128	85	43.2	0.3	0.278	40.00	3.20	0.022
5	138.67	128	86	41.6	0.3	0.231	32.00	9.60	0.069
6	133.33	128	88	40	0.3	0.200	26.67	13.33	0.100
7	128.00	128	90	38.4	0.3	0.179	22.86	15.54	0.121
8	122.67	128	91	36.8	0.3	0.163	20.00	16.80	0.137
9	117.33	128	93	35.2	0.3	0.152	17.78	17.42	0.148
10	**112.00**	**128**	**94**	**33.6**	**0.3**	**0.143**	**16.00**	**17.60**	**0.157**
11	106.67	128	96	32	0.3	0.136	14.55	17.45	0.164
12	101.33	128	98	30.4	0.3	0.132	13.33	17.07	0.168
13	96.00	128	99	28.8	0.3	0.128	12.31	16.49	0.172
14	90.67	128	101	27.2	0.3	0.126	11.43	15.77	0.174
15	85.33	128	102	25.6	0.3	0.125	10.67	14.93	0.175
16	80.00	128	104	24	0.3	0.125	10.00	14.00	0.175

Tables like this already have a long history. Von Böhm-Bawerk (1888) used them for instance to illustrate the optimal choice of production processes yielding the highest profit when the wage rate is given.

Figure 1 gives an overview of all possible steady states, shown in table 3, with production and consumption expressed per unit of labour and compared with both the age of the oldest capital in use and the investment level per unit of labour.

Figure 1 Production and consumption per labour unit in a two-sector economy



The left panel shows how the production of the consumption sector initially rises when replacement of machinery is postponed, but also how it starts to drop after its optimum at a replacement at 10 years. Besides it shows the steadily decreasing marginal product of labour on the oldest machinery in operation, until it reaches the wage rate which is equal to 1. The right panel starts with a marginal product equal to 1. The production of consumption commodities goes up with the investment level per unit of labour in the consumption sector until this level reaches a value of 0.17. Thereafter, a further rise of the investment level is accompanied by lower consumption.

In accordance with the Solow growth model figure 1 suggests that an optimal use of the production technology should obey the Golden Rule of Accumulation and coincides with Marshallian production if and only if the reward of labour equals the total net production. But in reality we hardly see ever increasing labour income shares that leaves less and less room for consumption to the owners of capital. On the contrary, recent decades rather show an increase in capital income shares from country to country. By shortages in the supply of physical capital the profit rates are frequently above zero. To explain these developments we first must see how in a world of Marshallian production some, but not all, firms may start to produce more efficiently by choosing their own investment strategies. By their higher levels of efficiency and profitability they get market power over the firms that keep producing in accordance with the neoclassical principles of equalizing marginal costs and marginal revenues. As we will see, the relatively high profit rates linked up with Marshallian production do not hamper the emergence of Schumpeterian firms, even if the capital income rates on their invested capital stick at lower values.

5. EFFICIENT PRODUCTION

The last paragraph gives some colour to Schumpeterian production, but also points to a resemblance with the Solow-growth model. By this model we know that raising the investment level per unit of labour could bring us to a situation in which, according to the Golden Rule of Accumulation, the net output per unit of labour reaches a maximum. In the Solow-growth model this situation will also be a Marshallian equilibrium with a capital income equal to zero.

By investing more from their revenue as supplier of labour and capital households may contribute to a more optimal relation between capital and labour. However, as the supply of capital relative to labour rises, labour would gain from its higher marginal productivity, while the capital income rate diminishes.

So, it seems that the owners of capital do not have much interest in supplying more capital. One should yet be aware of the simplifications behind the Solow-growth model, which only looks at a stylized relationship between the production factors labour and capital, without taking into account for instance the driving forces towards more efficient production as turn out from the production processes that succeed each other. The growth of total production should by analysed in relation to a supply of labour over time and an ever changing capital formation.

So, before looking at the emergence of Schumpeterian firms we should know how to identify efficient production. We investigate efficient production in the context of the sets that describe the actual production processes against the background of the development of all possible production processes. First, we are going to prove that positive profits over a sequence of periods implies that the production technology does not remain constant over time but develops as a result of an ever changing productivity. By wear and tear the productivity may diminish as capital becomes older, but the renewal of capital frequently implies on balance a steady increase in productivity[3].

[3] Productivity may rise as new capital asks for less labour and/or less intermediary input to produce one unit output. However, productivity of new capital may also become less, for instance if it takes more effort to win additional quantities of oil. The price of oil will then be determined by the new well that asks for much more effort and capital.

LEMMA 2:

Each sequence of Marshallian equilibria with profits that remain positive, has a production technology that changes over time, induced by the arrival of new or renewed production processes that are more or less productive in comparison with the other still profitable production processes.

Proof:
Among all profitable production processes we define the set that contains the processes that produce commodity *i* with the highest profit:

$$T_{ki\,MAX}^{t} = \{(\lambda_i^t, x_i^t, y_i^t) \in T_{i+}^t | \forall(w^t, p^t) \in P^t \; and \; \exists k \; : \; MAX(p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t)\}$$

Suppose that the production technology does not change over time. This would mean that new or renewed production processes for commodity *i* should always be equally profitable as the existing production processes. In that case, the set $T_{ki\,MAX}{}^{t}$ should also be equal to the set with the least profitable production processes of $T_{i\,MAX}{}^{t}$, which is:

$$LPT_{i\,MIN}^{t} = \{(\lambda_i^t, x_i^t, y_i^t) \in T_{i\,MAX}^t | \forall(w^t, p^t) \in P_i^t \;\; and \;\; \exists k: \;\; MIN\;(p^t(y_{ki}^t - x_{ki}^t) - w^t\lambda_{ki}^t)\}.$$

From lemma 1 we know that a Marshallian wage w^t approaches to the marginal product of labour $MPL_i{}^t$ if and only if the profit of the least profitable production process approaches to zero. So, as we have a sequence of Marshallian equilibria, all production processes would have a zero profit in case of a production technology that does not change over time. By contradiction, a positive profit implies differences between the sets $T_{ki\,MAX}{}^{t}$ and $LPT_{i\,MIN}{}^{t}$, which implies a changing production technology over time.
Q.E.D.

Next we ask ourselves to whom the benefits of the increased production are due. Are productivity gains as a consequence of a changing production technology over time gathered by the suppliers of labour or those of capital? In a simple model it may clear that labour gains in case of a rising amount of capital per unit of labour from a rising marginal productivity of labour. But in reality we have seen expanding production technologies with rising investment levels frequently accompanied by a rising labour supply, for instance as a result of labour migration from rural areas to industrial cities. Frequently we also see varying levels of unemployed labour supply. The dynamic interactions between these developments make the answer to who may claim the benefits of a steadily or irregular increasing production less obvious. We should therefore in principle reckon with all kind of paths along which societies can develop. However, for the sake of simplicity we leave profits that consist of rent income out of consideration. Rent income may originate for instance from the scarcity of fertile land or of raw materials such as oil.

So, we start with a certain point in time. As investments determine the development of capital over time, they may also determine to a large extent how the production technology set develops over time. To be able to account for production processes that change over time we focus on capital income available for consumption by capital owners as defined by $p^t\,(y_i{}^t - x_i{}^t - i_i{}^t) - w^t\,\lambda_i{}^t$, $(w^t, p^t) \in P^t$, instead of profit $p^t\,(y_i{}^t - x_i{}^t) - w^t\,\lambda_i{}^t$. Moreover, we redefine the supply set of profitable produced net output $Y_{i+}{}^t$ by replacing $y_{ki}{}^t - x_{ki}{}^t$ with $y_{ki}{}^t - x_{ki}{}^t - i_{ki}{}^t$ for all production processes involved in the production of each commodity *i* in a finite period *t*, *t = 0,…, N*:

$$Y_{i+}^t = \{(y_i^t - x_i^t - i_i^t, \lambda_i^t) | \sum_k(\lambda_{ki}^t, x_{ki}^t, y_{ki}^t) \in T_{i+}^t \; and \; C_i^t = \psi_i(C_i^{t-1}, i_i^{t-1})\}.$$

This redefinition, evidently, also affects the set $Y_+{}^t$ as follows from:

$$Y_+^t = \cup_i Y_{i+}^t.$$

Within the set Y_+^t we could discern a whole variety of feasible paths with the same starting point in order to elucidate efficiency.

DEFINITION 6

A finite sequence of production vectors $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$ of economy $\mathcal{E}$ is called a feasible path of horizon N if the pairs of two consecutive terms belong to Y_+^t, *i.e.,*

$$((y^t - x^t - i^t, \lambda^t), (y^{t+1} - x^{t+1} - i^{t+1}, \lambda^{t+1})) \in Y_+^t \ (t = 0, 1, \dots, N\text{-}1).$$

Let $Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ be the set of feasible paths of horizon N, all starting with the same initial production vector $(y^0 - x^0 - i^0, \lambda^0)$, made possible by the same initial capital C^0, and all bounded by a limited labour supply. Then we could define efficient production.

DEFINITION 7

Let $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$, $\{yy^t - xx^t - ii^t, \lambda\lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda\, l^0)/C^0, N)$. Then, $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$ is said to be more efficient than $\{yy^t - xx^t - ii^t, \lambda\lambda^t\}_{t=0}^N$ if $(y^N - x^N - i^N) \geq (yy^N - xx^N - ii^N)$, $(y^N - x^N - i^N) \neq (yy^N - xx^N - ii^N)$ and $\lambda^N \leq \lambda\lambda^N$.

DEFINITION 8

$\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$ is said to be efficient if there is no more efficient path in $Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ than $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$.

Nikaido (1968) uses this kind of definitions to prove his theorems of self-sustained efficient growth in economies endowed with production technologies that are characterized by convex cones. Although we do not have to rely on these restrictions, we eagerly follow his ideas on efficient production, which enable us to see that only by exception Marshallian production will be efficient. But first of all, one should note that efficient production is independent of the prices that market parties are willing to pay. Nevertheless, prices, relative prices, could help identify efficient production. Ricardo (1823a, b) tried to infer from their changes how the production technology has changed over time. He was convinced that in one way or another it must be possible to derive from the development of relative prices whether the physical production circumstances of a particular commodity are changed or not.

Here, we confine ourselves to the more simple relations between prices and production, whether or not efficient, for which we have to make the following assumptions.

ASSUMPTION 7: *The initial production vector $(y^0 - x^0 - i^0, \lambda^0) \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ involves Marshallian firms in the production of each commodity i and has a wage-price vector $(w^0, p^0) \in P^0$ which shows that all firms iM have a capital income $p^0\,(y_{iM}{}^0 - x_{iM}{}^0 - i_{iM}{}^0) - w^0\,\lambda_{iM}{}^0 > 0$.*

ASSUMPTION 8: *Each term of a feasible path $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ could reproduce itself: i.e. there exists a consecutive term $(y^{t+1} - x^{t+1} - i^{t+1}, \lambda^{t+1})$ with $\lambda^{t+1} = \lambda^t$, $i^{t+1} = i^t$ and $y^{t+1} - x^{t+1} \geq y^t - x^t$.*

Assumption 8 is not quite realistic. It excludes, for instance, the possibility that shocks affect the production technology. Reckoning with shocks like fire, storm and other calamities would substantially complicate the proof of theorem 3 whilst it delivers only the notion that after such shocks the reproduction capabilities need to be restored. In addition it also excludes the possibility of exhaustible resources that asks for more labour in order to be able to reproduce the existing production. Reckoning with this kind of possibilities would equally request a much more complicated argumentation. In fact by assumption 8 we merely assume that the production technology smoothly changes over time.

To be as clear as possible we confine ourselves to feasible paths that are stylized by the assumptions 7 and 8. In doing so we are going to search for efficient production among all (stylized) feasible paths with consecutive terms that are all characterized by profitable production for the wage-profit-price vectors that belong to P^t.

THEOREM 3 (Golden Rule of Accumulation)

Given any wage-price vector $(w^t,p^t) \in P^t$, there exits an efficient path $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ that maximizes $(p^t (y^N - x^N - i^N) - w^t \lambda^N)$. This maximum coincides with Marshallian production for each commodity i if and only if wage $w^t = (p^t (y^N - x^N - i^N) / \lambda^N$. Profit rate $\pi_{rate}{}^t$ will then be equal to $p^t i^N / V^t$ which implies a capital income equal to zero.

Proof:
All consecutive terms belonging to a feasible path $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ consist of production processes $(\lambda_{ki}{}^t, x_{ki}{}^t, y_{ki}{}^t) \in T_{i+}{}^t$. As all $T_{i+}{}^t$ are compact by assumption 2, $Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ is also compact. So, for any $(w^t,p^t) \in P^t$ the function $(p^t (y^t - x^t - i^t) - w^t \lambda^t)$ has a maximum in $p^t (y^N - x^N - i^N) - w^t \lambda^N$. This maximum clearly belongs to an efficient path. If there would be a more efficient path $\{yy^t - xx^t - ii^t, \lambda\lambda^t\}_{t=0}^N$, then $(p^t (yy^N - xx^N - ii^N) - w^t \lambda\lambda^N) > (p^t (y^N - x^N - i^N) - w^t \lambda^N)$ because $(yy^N - xx^N - ii^N) \geq (y^N - x^N - i^N)$, $(yy^N - xx^N - ii^N) \neq (y^N - x^N - i^N)$ and $\lambda\lambda^N \leq \lambda^N$. This would contradict that $p^t (y^N - x^N - i^N) - w^t \lambda^N$ gives a maximum.

In order to have Marshallian production for each commodity *i* as well the profit function $(p^t (y^t - x^t) - w^t \lambda^t)$ must also reach a maximum for the same $(w^t,p^t) \in P^t$. Moreover, this maximum must be sufficient to reward the owners of the capital in such a way that they are able to maintain their capital, so $(p^t (y^t - x^t) - w^t \lambda^t) \geq p^t i^t$.

Suppose that the production of one commodity is not Marshallian. An additional unit of labour would then enable a higher profitable production. Let $(p^t (y^M - x^M) - w^t \lambda^M)$ denote this maximum of the profit function for the production of all commodities together with an accompanying investment i^M.

As we have already seen, $(y^M - x^M - i^M, \lambda^M)$ cannot be more efficient than $(y^N - x^N - i^N, \lambda^N)$. However, $(y^M - x^M - i^M, \lambda^M)$ could indeed be less efficient with lower wages and more labour supply. But not if $w^t = (p^t (y^N - x^N - i^N) / \lambda^N$. In that case $(y^M - x^M - i^M, \lambda^M)$ must be equal to $(y^N - x^N - i^N, \lambda^N)$, otherwise the profit function of all commodities together would result in a negative profit which contradicts Marshallian production. If the production is both efficient and Marshallian the profit function reaches a maximum of zero for $w^t = (p^t (y^N - x^N - i^N) / \lambda^N$. By substituting w^t the profit function can be rewritten as $(p^t i^N - \pi_{rate}{}^t V^t)$. This function cannot be positive otherwise there would be undistributed profits. A negative value is also prohibited, because profits are necessary to enable the reproduction of the efficient production by assumption 8. So, profit rate $\pi_{rate}{}^t$ must be equal to $p^t i^N / V^t$.
Q.E.D.

Clearly, a production technology could give rise to a variety of efficient paths, all characterized by a binding input of labour. But by assumption 7 all these paths that may become efficient in the end, are in the beginning still characterized by positive capital incomes. So, in the beginning capital is still the relative scarce production factor. Among all feasible paths some of them may early begin to show how more capital relative to labour paves the way for more efficient production. But as long as the consecutive terms still have positive capital incomes further progress towards efficient production can be made. Although we hardly see that the economy as a whole reaches efficient production in reality, we do see a lot of activities that go into that direction. To be able to describe these activities in more detail, we need first an additional definition (given definition 7 a rather trivial definition) to compare intermediate consecutive terms of feasible paths at the same time.

DEFINITION 9
Let $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$, $\{yy^t - xx^t - ii^t, \lambda\lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$. Then, $(y^t - x^t - i^t, \lambda^t) \in \{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$ is said to be more efficient than $(yy^t - xx^t - ii^t, \lambda\lambda^t) \in \{yy^t - xx^t - ii^t, \lambda\lambda^t\}_{t=0}^N$ if $(y^t - x^t - i^t) \geq (yy^t - xx^t - ii^t)$, $(y^t - x^t - i^t) \neq (yy^t - xx^t - ii^t)$ and $\lambda^t \leq \lambda\lambda^t$.

The following lemma shows that from each consecutive term of a feasible path with a positive capital income a variety of feasible paths can originate that all differ in efficiency.

LEMMA 3

For each term of a feasible path $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ and a wage-price vector $(w^t,p^t) \in P^t$ that implies Marshallian production for each commodity *i* and a capital income $p^t (y^t - x^t - i^t) - w^t \lambda^t > 0$ there exists a consecutive term $(yy^{t+1} - xx^{t+1} - ii^{t+1}, \lambda\lambda^{t+1})$ with $p^t i^t < p^t ii^{t+1} \leq p^t (y^t - x^t) - w^t \lambda^t$ that is more efficient than the consecutive term $(y^{t+1} - x^{t+1} - i^{t+1}, \lambda^{t+1})$ that only reproduces the original term $(y^t - x^t - i^t, \lambda^t)$.

Proof:
Let $Y((y^t - x^t - i^t, \lambda^t)/C^t, N)$ be the set of feasible paths of horizon *N*, all starting with the same initial production vector $(y^t - x^t - i^t, \lambda^t) \in Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$, the same initial capital C^t and all having a consecutive terms $(y^{tt} - x^{tt} - i^{tt}, \lambda^{tt})$ for which holds $(y^{tt} - x^{tt} - i^{tt}) \geq (y^t - x^t - i^t)$ and $\lambda^{tt} \leq \lambda^t$ for all *tt* from *t* to *N*. This set is clearly not empty, because $(y^t - x^t - i^t, \lambda^t)$ could reproduce itself until horizon *N* by assumption 8. The set is also compact, because $Y((y^0 - x^0 - i^0, \lambda^0)/C^0, N)$ is compact as we have seen earlier. Moreover we know that at least in *t+1* capital C^{t+1} has benefitted not only from investment i^t but also from an extra investment out of the positive capital income $p^t (y^t - x^t - i^t) - w^t \lambda^t$. As long as the capital income remains positive extra investment out of this income may occur. In the meantime i^{tt} will gradually rise in order to comply with assumption 8.

So, for any wage-price vector (w^t,p^t) the profit function $(p^t (y^t - x^t - i^t) - w^t \lambda^t)$ reaches a maximum within $Y((y^t - x^t - i^t, \lambda^t)/C^t, N)$. By theorem 3 we know that this maximum belongs to an efficient path. If the extra investments out of the capital income lead to a drop in this income to zero, we also know that this maximum coincides with Marshallian production.

Therefore, any possible path resulting from extra investment must be more efficient than the feasible path that only reproduces $(y^t - x^t - i^t, \lambda^t)$, otherwise $(y^t - x^t - i^t, \lambda^t)$ would already be efficient which contradicts its Marshallian production with a positive capital income. The efficient path, resulting in $(y^N - x^N - i^N, \lambda^N)$ and a capital income that has dropped to zero, uses more capital than C^t. If C^N would be equal to C^t, the more efficient production of year *N* with a capital income equal to zero must already be possible in year *t*. This again contradicts the Marshallian production with a positive capital income of the feasible path.

In year *t+1* capital C^t increases, because $C^{t+1} = C^t + i^t + \Delta$ with $p^t i^t < p^t (i^t + \Delta) \leq p^t (y^t - x^t) - w^t \lambda^t$. Although C^{t+1} could still be less than C^N the increased capital enables a consecutive term that is more efficient than $(y^t - x^t - i^t, \lambda^t)$.
Q.E.D.

The efficient production, as identified until now with definitions 7, 8 and 9, concerns the production of all production processes together. We eagerly would like to use similar definitions to identify whether a firm is more efficient than another firm. However, it should be noted that that output y^t of each production vector $(y^t - x^t - i^t, \lambda^t)$ belonging to a feasible path includes both intermediate consumption x^t and investment i^t. In fact each production vector $(y^t - x^t - i^t, \lambda^t)$ represents the net output produced by labour λ^t provided with capital C^t. The all-encompassing view of the aggregate level of economy $\mathcal{E}$ enables us to identify efficient production independently of wages and prices

On a firm level we do not have such an all-encompassing view. In producing the same commodity *i* firms could use different intermediate consumption or different machines, which makes a comparison of merely physical production processes rather indecisive. Some firms, producing automobiles for instance, may produce the necessary cylinder blocks themselves. Other automobile factories may buy them from firms that only produce cylinder blocks. So, if we would like to know (independently of wages and prices) which automobile factory is more or most efficient, we should also take into account the firms that deliver cylinder blocks. Reckoning with other inputs as well will bring us in no time to the aggregate level that encompasses all production processes. If we know that among all firms only automobile factories differ in efficiency, we could infer from the resulting almost equal feasible paths which firms produce automobiles more or most efficiently in comparison with other firms.

However, although efficient production is ultimately independent of wages and prices, wages and prices do matter in identifying firms that have managed to produce more efficiently than other firms. Like Ricardo, who was curious to derive from relative price developments whether new production processes were implemented that enable to produce more efficiently than before, we also make use of the prevailing wages and prices in our search for more efficient firms. But first and for all we must find out which firms succeed in exploiting the existing production technology set, that changes over time, more efficiently than the surrounding Marshallian firms. By confining ourselves to the existing production technology we can focus on the fundamentals of oligopolistic competition and see how this competition emerges independently of wages and prices.

6. SCHUMPETARIAN PRODUCTION EVOKES OLIGOPOLISTIC COMPETITION

From paragraph 5 it turns out that uncountable feasible paths exist that all differ in efficiency. Moreover, there are always more efficient paths in comparison with the feasible paths characterized by Marshallian production with a positive capital income. More investments may offer firms transitions to more efficient paths. But if those firms keep using their enlarged capital, enabled by an enlarged input of labour, up to its maximum profit potential, Marshallian production remains in place. Wages and prices will then continue to reflect the balance between costs and revenues as shown by the least profitable production processes.

Schumpeterian firms do not have the drive of Marshallian firms to maximize profits. Instead they are interested in producing as efficiently as possible. These firms do not bother with all the effort that is necessary to keep old machinery in operation as long as it remains profitable. The drive to produce as efficiently as possible frequently arises in firms that also produce to a large extent the capital they need for their final production themselves. At the time that the production of automobiles became part of the production technology many firms entered the automobile markets. But the ‘perfect’ competition, initially shown by these markets, changed rather rapidly into oligopolistic competition, driven by not less but more efficient production. The automobile sector showed how this was possible within the same, for all firms accessible production technology set, characterized by productivity changes that only allow to determine whether a particular production process in view of the prevailing wages and prices is still profitable.

Although Schumpeterian firm do not strive for a maximum profit as Marshallian firms do, we will see that they do succeed in attaining a capital income per unit of production that, by their implicit higher efficiency, is higher than the capital income per unit of production realized by Marshallian firms. This relative higher profit gives the Schumpeterian firm market power over Marshallian firms. The following definition resembles the specific market power that Marshallian firms have over each other by definition 2.

DEFINITION 10
Firms *iS,* producing commodity *i*, have market power over Marshallian firms *iM* that produce the same commodity *i,* if, under the prevailing wage-price vector $(w^t,p^t) \in P^t$, their capital income per unit of

production $(p^t (y_{iS}^t - x_{iS}^t - i_{iS}^t) - w^t \lambda_{iS}^t) / y_{iS}^t$ is higher than $(p^t (y_{iM}^t - x_{iM}^t - i_{iM}^t) - w^t \lambda_{iM}^t) / y_{iM}^t$.
DEFINITION 11
A firm that produces commodity *i* is called a Schumpeterian firm if it has market power over one or more Marshallian firms that produce the same commodity *i.* The production of Schumpeterian firms is called Schumpeterian production.

With definition 10 we are able to identify the Schumpeterian firms in the set of firms that produce more efficiently than Marshallian firms. This set is clearly not-empty as follows from lemma 3. With theorem 4 we formally prove the emergence of oligopolistic competition.

THEOREM 4 (Oligopolistic competition)

Let $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0) / C^0, N)$ be a feasible path of economy $\mathcal{E}$ in which, given a wage-price vector $(w^t,p^t) \in P^t$, Marshallian firms *iM* produce commodity *i* with a capital income $p^t (y_{iM}^t - x_{iM}^t - i_{iM}^t) - w^t \lambda_{iM}^t > 0$. Among those firms Schumpeterian firms *iS* may stand up that get market power over Marshallian firms: i.e. they are able to influence wages and prices, although these wages and prices keep reflecting the balance between costs and revenues as shown by the least profitable production processes of Marshallian firms.

Proof:
By lemma 3 we know that for each term of $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N \in Y((y^0 - x^0 - i^0, \lambda^0) / C^0, N)$ there exists a consecutive term $(yy^{t+1} - xx^{t+1} - ii^{t+1}, \lambda\lambda^{t+1})$ that by its extra investments out of the positive capital income is more efficient than $(y^{t+1} - x^{t+1} - i^{t+1}, \lambda^{t+1})$ that only reproduces the original term $(y^t - x^t - i^t, \lambda^t)$. Evidently, on an aggregate level more efficient production implies a higher net output and may also result in a higher capital income. For it is clear by definition 9 that $(p^{t+1} (yy^{t+1} - xx^{t+1} - ii^{t+1}) - w^{t+1} \lambda\lambda^{t+1}) > (p^{t+1} (y^{t+1} - x^{t+1} - i^{t+1}) - w^{t+1} \lambda^{t+1})$.

As we know that all production of the consecutive terms belonging to $\{y^t - x^t - i^t, \lambda^t\}_{t=0}^N$ can be attributed to the firms that form part of the feasible path, we also know that the consecutive term $(yy^{t+1} - xx^{t+1} - ii^{t+1}, \lambda\lambda^{t+1})$ may contain a not-empty set of firms *iS* that start producing more efficiently commodity *i* than it would be in $(y^{t+1} - x^{t+1} - i^{t+1}, \lambda^{t+1})$ and also more efficient than the Marshallian firms *iM* that keep producing commodity *i* in $(yy^{t+1} - xx^{t+1} - ii^{t+1}, \lambda\lambda^{t+1})$ in a similar way as they did in $(y^t - x^t - i^t, \lambda^t)$. These firms *iS* will also share the extra capital income as a result of the more efficient production. As long as Marshallian production remains in place there will be a wage-price vector that continues to reflect the balance between costs and revenues of the least profitable production processes. So, (w^{t+1},p^{t+1}) remains unchanged in case of such a transition to a more efficient consecutive term.

In order to prove that the more efficient firms *iS* are Schumpeterian firms we only have to show that $(p^{t+1} (yy_{iS}^{t+1} - xx_{iS}^{t+1} - ii_{iS}^{t+1}) - w^{t+1} \lambda\lambda_{iS}^{t+1}) / yy_{iS}^{t+1}$ is higher than $(p^{t+1} (yy_{iM}^{t+1} - xx_{iM}^{t+1} - ii_{iM}^{t+1}) - w^{t+1} \lambda\lambda_{iM}^{t+1}) / yy_{iM}^{t+1}$. Because the more efficient firms have a higher capital income, this inequality holds if we a firm *iS* compare with a firm *iM* that produces the same quantity of production yy^{t+1}.

Each firm *iS* may use its higher capital income to take over labour from Marshallian firms by offering a wage that is higher than the prevailing wage rate w^{t+1}. If a rising labour supply does not provide the Marshallian firm with sufficient replacement, the wage rate will remain on a higher level by which a part of the production of Marshallian firms is no longer profitable. The Schumpeterian firms may use the acquired labour to expand their production and so enhance the efficiency of the total economy. Schumpeterian firms have also the power to use their higher capital income to ask lower prices for the commodities they produce in comparison with the prevailing prices p^{t+1}. This will also force the Marshallian firms to an adjustment because a part of their production again becomes no longer profitable. So, Schumpeterian firms may use their power to influence wages and prices and thereby

force Marshallian firms to adapt their production in such a way that the influenced wages and prices remain reflecting the balance between of costs and revenues as shown by the least profitable production processes of Marshallian firms.
Q.E.D.

In comparison with earlier stages of the economy in which households by their supply of capital determine the velocity at which the production possibilities of the evolving production technology are exploited, we are now able to see how firms in more advanced stages play a more decisive role in economic development. Especially firms owned and managed by the capital provider may become aware of more efficient methods of production, more efficient in comparison with the traditional Marshallian firms that keep production processes in operation as long as they remain profitable. But other firms as well may embrace the opportunity to produce more efficiently and, for instance, start to issue shares by which the suppliers of capital express their confidence in sharing the risks taken by the firm. By their higher efficiency Schumpeterian firms get market power over Marshallian firms.

The resulting oligopolistic competition enables Schumpeterian firms to become forceful drivers of economic development. By their higher profits per unit of production they are not only capable to expand their market share at the expense of Marshallian firms, but they may also invest their profits in research by which the boundaries of the production technology may be pushed forward. So, Schumpeterian firms frequently operate at the cutting edge of both efficient production and renewal of production by the introduction of new products and production processes. Markets that are initially characterized by many participating firms may therefore rapidly develop into markets with only a few firms. However, the thread of falling behind is always present. A once Schumpeterian firm may therefore ever again become part of the traditional Marshallian firms.

Marshallian firms remain of importance. Their production processes frequently determine the wages and prices on the markets that reflect the balance between costs and revenues on the least profitable production processes. The interaction between Schumpeterian and Marshallian firms has manifold manifestations. Without Marshallian firms Schumpeterian firms would not be able to make extra profits per unit of output and to become drivers of a prosperous economic development. As Schumpeterian firms expand and take over labour from Marshallian firms, the wages determined by Marshallian production may also rise. At the same time the arrival of new products and production processes will frequently be accompanied by firms that keep the old production processes in operation as long as they remain profitable. An increasing prosperity will therefore frequently be characterized by graduality.

However, prosperous developments could also easily become at stake. The equilibrating forces, assumed by the neoclassical theory, have in the reality of oligopolistic competition only a limited range of action. They just demonstrate themselves at the least profitable production processes of Marshallian firms. If only Marshallian firm would be present, then of course demand and supply of labour would ultimately lead to equilibrium wages that equal demand and supply. But as soon as Schumpeterian firms are active, the realization of this kind of equilibria is no longer guaranteed. If the employment of Marshallian firms diminishes by the expansion of Schumpeterian firms and unemployment starts to rise, one could try to reverse this process by wage moderation. But Schumpeterian firms that use their market power and respond with lower prices may then hinder the effectiveness of wage moderation. Manifold interactions could occur. In case of a widespread cyclical downturn Schumpeterian firms may simply react by consolidating their market position and preparing themselves for an intensified competition with other Schumpeterian firms. A prolonged period of unemployment may then result. On the other hand, if they expect a quick recovery they may be willing to further expand their market share and so contribute to the fulfilment of their expectations.

The interactions between Schumpeterian and Marshallian firms become even more complex if Schumpeterian firms in one country have market power over Marshallian firms in several other countries as well. Worldwide operating Schumpeterian firms have frequently different market strategies for different countries induced by the differences in competition they face from Marshallian firms in one country to another or even also from their Schumpeterian competitors.

The complexity of possible interactions increases further if we reckon with different currencies for different countries. From a neoclassical background it is understandable that one thinks that firms of a country see their export opportunities increase if their currency devaluates. But in the reality of oligopolistic competition these expectations may be wrong. Schumpeterian firms operating from a country with a strong currency could even benefit from a further rise in the value of their currency. Without any action their profits may increase not only because they benefit from lower import prices. Profits may also go up relatively if their foreign prices will remain closely linked to their home prices. The customers in the devaluating countries will then have to pay a larger share of their income to these Schumpeterian firms. If these customers have difficulty in paying higher prices, the Schumpeterian firms that export from countries with a raising currency may prevent a threatening loss of market share by using their increased profits to adjust their foreign prices to the prices of the foreign competitors or even further if they would like to expand their market share.

Governments should not rely on neoclassical theories either. Although in some circumstances a policy of promoting wage moderation could be helpful to combat long lasting unemployment, there is no guarantee for success. The same holds for policies to contain the market power of Schumpeterian firms. These could easily have even adverse effects. All depends on the specific situation on the relevant markets. In the Netherlands Prime Minister Lubbers fought high unemployment in the 1980's with a policy of wage moderation strongly supported by investment stimuli. These stimuli were so successful in modernizing the economy and finding the way back to full employment that they had to be ended by their own success. There are more examples of political leadership showing that, with the benefit of hindsight, circumstances were rightly assessed and the right measures were pushed through.

There is a kind of similarity between entrepreneurial and political leadership. Schumpeterian firms cannot but emerge from entrepreneurial capabilities. Entrepreneurs intuitively see the opportunities offered by the ever changing set of production possibilities and know how to produce more efficiently than the Marshallian producers who think to attain maximum profits. And, indeed, on the level of separate firms the most profitable production that can be derived from wages and prices is Marsallian production. The most efficient production, however, cannot be derived from wages and prices, but has to be determined on the level of all firms together. Only knowledge of the development of the total physical production possibilities enables to determine the most efficient production. Schumpeterian firms emerge because of entrepreneurs who have a thorough knowledge of the production possibilities and see attainable efficiency that goes beyond the efficiency as can be derived from market information.

Political leadership asks for similar capabilities. Successful policies are adapted to the interaction between Schumpeterian and Marshallian firms and induce an optimal use of the available production opportunities by prefer. However, for politicians it is equally not easy to see how the physical production technology develops over time and how the interaction between Schumpeterian and Marshallian firms could hamper a prosperous economic development. It strongly depends on the specific circumstances which bottlenecks have to be taken away.

At present, new technologies become rapidly available enabling a worldwide turn into a more sustainable development. The implementation of some of them will even brook no delay to prevent future problems. At the same time there is almost no country left that is able to give the right example alone. All developed countries face an increased dependency of international relations. These are more and more influenced by the rapid development of emerging countries. Their firms do not only put pressure on the price-competitiveness of especially the Marshallian firms in developed countries, but also have the ambition and the capabilities to develop themselves to real Schumpeterian firms with an enormous growth potential on their home markets. In the meantime, the stagnant real investment in developed countries creates flows in financial capital that repeatedly seek a way out in speculative bubbles with ever more uncertainty as a result. In view of these strongly interdependent relations it is difficult to overcome the present watershed that prevents us from entering a real sustainable world, although the needed political leadership may increasingly encounter societal support.

7. FUTURE WORK

The manifold interactions between Marshallian and Schumpeterian firms have important consequences for economic development and the policies by which governments could foster this development. These policies should by prefer stay in tune with the development of the production possibilities, and contribute to an optimal use of these possibilities by tackling bottlenecks as soon as they become obvious. Therefore, it is important to get knowledge on how the physical production technology develops over time, reckoning with the use of this technology by both Marshallian and Schumpeterian firms especially on a worldwide level.

Empirical work to describe the complex international relations could build upon a vintage model that allows determining the share of Schumpeterian firms in total production assuming that the technology sets of developed countries are more or less the same. From an application to the manufacturing industry of the US, UK, Germany, France and the Netherlands (Moons (1996)) it appears that especially in the US Schumpeterian firms have a relatively great share in manufacturing output, explaining the more resilient production in the US while other countries experience greater fluctuations in economic growth. The higher share of Schumpeterian firms in the US goes along with a higher wage share. Although fully in accordance with the Golden Rule of Accumulation, this higher wage share may also reflect a high degree of labour put into research to further develop the production technology.

This empirical work about the occurrence of oligopolistic competition in separate countries needs to be extended with a description of the cross border activities of firms which may further strengthen the position of Schumpeterian firms. The same holds for a more elaborate description of the business cycle and the way in which the interaction between Marshallian and Schumpeterian firms affects employment opportunities. But equally important is to deepen the theory by identifying the principles which reveal the developments of the set of possible production and the way it is used.

“When commodities varied in relative value, it would be desirable to have the means of ascertaining which of them fell and which rose in real value, and this could be effected only by comparing them one after another with some invariable standard measure of value, which should itself be subject to none of the fluctuations to which other commodities are exposed” (Ricardo (1821), p 43). The search for such an invariable measure confronted Ricardo with difficult problems, but this measure in fact asks for nothing else than how can we derive from market information of which commodities the physical production circumstances are changed by changes in (the use of) the set of possible production. The solution of this problem can be found by looking at the production of all commodities together. By only looking at separate commodities unsurmountable problems arise from the absence of a commodity of which the production circumstances never change, and even if such a commodity would exist, it still could not be a perfect measure of value because its value is also subject to changes in the division of the total production between the labourers and capital owners. Nevertheless, Ricardo remained of the opinion that “if we were in possession of the knowledge of the law which regulates the exchangeable value of commodities, we should be only one step from the discovery of a measure of absolute value” (Vol. IX, p. 377). Although the correspondence of Ricardo with Malthus gives ample reason to believe that they were close to a solution indeed, it must also be acknowledged that McCulloch let himself fall behind. The question of how to measure of which commodities the production circumstances have changed over time, he left to be settled by his masters. He only showed interest in the relative values of commodities in the same market at the same time. These follow from comparing the productivity of both labour and capital. By extending Ricardo's exposition on the diminishing rent of land to homogeneous capital he was a frontrunner in paving the way for the neoclassical production function.

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